# The Carry Trade and Uncovered Interest Parity when Markets are Incomplete*

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## Abstract

Many of the leading models of the carry trade imply that, contrary to the empirical evidence, a country's currency depreciates in times of high consumption and output growth, a manifestation of the Backus and Smith (1993) puzzle. We propose a modification of these models to account for financial market incompleteness and show that such a modification can induce positive correlation between currency appreciation and consumption or output growth while, at the same time, helping resolve the Backus and Smith (1993) and Brandt, Cochrane, and Santa-Clara (2006) puzzles. Furthermore, in many of the existing models, the assumed fundamental cross-country differences (output volatility, growth, and risk attitude) responsible for interest rate differentials also appear at odds with the data. We document that default risk and financial openness are strongly related to interest rate differentials and carry trade profits in the data. The incomplete markets model we propose is consistent with these novel empirical facts.



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# 1 Introduction

In a simple, risk neutral setting, uncovered interest parity (UIP) predicts that the currency of a country with a high interest rate is expected to depreciate so that the profit is the same as from investing in a low interest rate bond. Empirically, UIP fails and the expected profit from the carry trade (investing in a high interest rate bond abroad, and then converting back to the home currency) is positive.[1]

Academics have attributed the failure of UIP and the profitability of the carry trade to its risk. Several channels for this risk have been identified. These include cross-country differences in the volatility of output and consumption (Colacito (2009), Bansal and Shaliastovich (2012), Tran (2013)); risk aversion (Verdelhan (2010), Heyerdahl-Larsen (2014)); expected growth (Colacito and Croce (2013)); loading on aggregate productivity risk (Colacito, Croce, Gavazzoni, and Ready (2015)); size (Dumas (1992)[2], Hassan (2013), Martin (2013)), and exporter versus importer status (Ready, Roussanov, and Ward (2016)). Many of these models have also assumed that financial markets are complete.

Our contribution is twofold. First, we show that many of the leading economic models of the carry trade are at odds with the data along an important dimension: the correlation of currency appreciation (and therefore carry trade profit) of the target (high interest rate) country, with output and consumption growth in that country. It is positive in the data but negative in these models.[3] This correlation is closely related to the Backus and Smith (1993) puzzle[4] and to the Balassa (1964)

[1]Hassan and Mano (2015) argue that the failure of UIP and the carry trade may be distinct phenomena. In our model they are closely related.

[2]Dumas (1992) differs from much of the rest of the literature in that differences in size come about due to costs of shipping capital abroad. Since capital is an asset, as opposed to a consumption good, we believe that costs of financial trade, which is one of the channels we explore, are similar in spirit to the channel in Dumas (1992).

[3]The positive relationship between currency appreciation and relatively strong local economic growth is not only true empirically, but widely consistent with practitioner intuition and the financial press. For example Thomas (2014) in the *The New York Times* writes "Across trading desks in New York, London and elsewhere, analysts are rushing to raise their dollar forecasts based on the resurgence in the American economy. The recent rally in the dollar... underscores expectations that the United States economy will continue to grow at a faster clip than that of Europe, Japan and even large emerging markets."

[4]Complete markets imply that $m_H - m_F = \Delta e$. Here $m$ is the log of the stochastic discount factor and $e = \log(E)$, where $E$ is the amount of foreign currency per unit of home. Thus, $\Delta e > 0$ implies an appreciation of home currency. This relation can be derived through no arbitrage, see Backus and Smith (1993). For example, with CRRA preferences, $m = -\gamma \Delta c$, thus $\Delta c_H - \Delta c_F = -\frac{1}{\gamma}\Delta e$. The intuition is that after a positive productivity shock at home, the domestic price falls, implying a a currency depreciation. Rather than correlations, the Backus-Smith puzzle is usually stated in terms of second moments, i.e., countries with a volatile real exchange rate do not tend to have volatile real consumption. Although we do not focus on second moments, our model's implications are consistent with the data. In particular: across the different specifications of our model that we consider in Section 4, there is a positive relationship between exchange rate volatility and consumption growth volatility; within any model, exchange rate volatility is higher than consumption growth volatility.

and Samuelson (1964) effect.[5] We show that a modification of these models that allows for financial market incompleteness can generate correlations that are in line with the data, and at the same time, help resolve the Backus and Smith (1993) and the Brandt, Cochrane, and Santa-Clara (2006)[6] puzzles.

Second, we argue that many of the existing models have an additional potential shortcoming: the fundamental cross-country differences assumed responsible for interest rate differentials (and therefore carry trade profits) appear at odds with the data. Any model of the carry trade and UIP needs some kind of asymmetry – otherwise all countries would have identical interest rates and expected currency appreciation. We explore two alternative drivers of interest rate differentials and carry trade profits: cross-country differences in default probability, and in impediments to financial trade. Both channels rely on incomplete financial markets to deliver carry trade profits; both are supported by the data. Additionally, counter-cyclical inflation, arising endogenously in our model, can reinforce these channels and is a third source of carry trade profit. These channels have not been identified by the prior literature, either empirically or theoretically.[7]

To study these channels, we build a model of international trade, tradable and non-tradable goods, and incomplete financial markets. As will be discussed below, our mechanism is quite different from previous explanations of the carry trade, and our model leads to additional empirical implications. The model is general enough to study most of the aforementioned explanations of the carry trade in the same setting, which we do as part of the analysis.

Our model is not the first to study incomplete markets in an international setting. Most work has focused only on the Backus and Smith (1993) puzzle, that is explaining a positive relationship between currency appreciation and consumption growth. Benigno and Thoenissen (2008) employ the same two ingredients as us: two nominal bonds and tradable and non-tradable goods, though they do not allow for default. Corsetti, Dedola, and Leduc (2008) is a related model but with only one bond. Recently, Rabitsch (2014) explores the implications of incomplete markets for UIP. In her setup, borrowing constraints lead to imperfect risk sharing and to pro-cyclical movements in

[5]The Balassa-Samuelson effect is the empirical observation that countries which experience higher output growth also experience currency appreciation. A common way to rationalize this is through a model of tradable and non-tradable goods.

[6]This puzzle is explained in detail in Section 4.5.

[7]The impediments to financial trade channel is somewhat similar to limits to arbitrage, as in Gromb and Vayanos (2003) and Gabaix and Maggiori (2015), although neither paper focuses on UIP or the carry trade.

a country's currency. However, as there is only one bond, the carry trade is not possible in her economy.

Our paper is also closely related to Gabaix and Maggiori (2015), who provide a general framework of international trade and exchange rates when financial markets are incomplete. Gabaix and Maggiori (2015) focus on limits to arbitrage preventing financial intermediaries from perfectly redirecting capital towards where yield is highest. One of the frictions we consider - the cost of trade - has a flavor of the Gabaix and Maggiori (2015) channel, although others - such as the limits to the types of securities traded due to the possibility of default - are distinct. More importantly, although Gabaix and Maggiori (2015) show that market incompleteness may matter for the carry trade, the carry trade is not a focus of their analysis and they do not touch on the fundamental reasons for the interest rate differentials, carry trade profits, or covariance of these profits with fundamentals.[8] We, on the other hand, provide a more detailed analysis of various potential explanations of the carry trade and how they interact with incomplete markets; we also link our theoretical results to the data.

Although most complete markets models (counterfactually) imply a negative relationship between consumption growth and currency appreciation, one exception is the complete markets economy of Colacito and Croce (2013). In their model, like in other complete markets models, a positive (long run) productivity shock leads to a currency depreciation. However, the positive productivity shock also leads to lower (relative) consumption growth as the country saves more for precautionary reasons. As a result, like in our model and in the data, their model achieves a positive correlation between consumption growth differentials and currency appreciation. At the same time, due to the assumed cross-sectional differences in volatility and expected growth, the UIP does not hold in their model, as in the data. Although our model targets related phenomena, our mechanism is very different because in our model markets are incomplete and risk sharing is imperfect. Our framework differs in two important aspects from existing studies that rely on complete financial markets. First, in our model there is a *positive* correlation between currency appreciation (of the carry trade target country) and carry trade profit, productivity shocks, output growth, and consumption growth. Second, to explain violations of UIP and profitability of the carry trade we do not need to rely on asymmetries in expected growth rates, or expected volatility across countries.

[8]For example, in the only application of their model to the carry trade, they simply assume interest differentials are due to discount rate differentials.

The rest of the paper is laid out as follows. Section 2 reviews existing empirical evidence regarding the carry trade, presents new empirical results, and relates these results to past models of the carry trade. Section 3 describes our model. Section 4 shows why incomplete markets can help explain the empirical behavior of exchange rates and interest rates. Section 5 concludes. Appendix A compares our model to complete market models that rely on asymmetry in output volatility and risk aversion as sources of carry trade profits. Appendix B contains details of the model solution and Appendix C describes the data used in our empirical analysis.

## 2 Empirical evidence

Many different explanations of the carry trade and the failure of UIP have been proposed. To tell apart which are the most relevant for the real world, it is important to compare the assumptions and implications of these explanations to the data. To do so, in this section we examine various empirical properties of exchange rates, interest rates, and the carry trade. The empirical results in this section focus on one of two questions: what are currency returns correlated with? and what drives interest rate differentials across countries?

The first important distinction between many of the previous explanations and the model we propose below has to do with the correlation of currency returns with macroeconomic quantities. In particular, most complete markets models imply a *negative* correlation of realized currency appreciation (or of realized carry trade profits), with fundamentals such as realized consumption growth or realized output growth. In the data, and in the model we develop below, this correlation is *positive*.

Second, most models of the carry trade require particular differences in size, loading on world risk, or volatility between funding (low interest rate) and target (high interest rate) countries. We will empirically explore these differences below. We show that many of the cross-country differences identified by past models do not appear related to interest rate differentials or to the carry trade. We identify three fundamental differences that do seem to matter, both empirically, and later in our model: differences in inflation volatility, differences in financial market openness, and differences in sovereign credit risk.

A third distinction between our model and much of the previous work is the type of exogenous shock affecting a model. Though we do not explore this empirically, Stockman and Tesar (1995), and Drozd and Nosal (2009) have shown that shocks to tradable output are 1.5 to 2.5 more volatile than shocks to non-tradable output. However, most explanations of the carry trade have focused on non-tradable shocks. As will be shown below, the type of shock is crucial for the exchange rate behavior of most models. For this reason, as discussed below, our model focuses on tradable shocks.

Note that in this paper the notation is always such that $E$ is the nominal amount of foreign currency per unit of home currency. Lower case letters indicate logs. Therefore, $E_{t+1}/E_t > 1$ or $e_{t+1} - e_t = \Delta e_{t+1} > 0$ means that the Home currency (in this case the U.S. dollar) appreciates against the Foreign currency. In some of the tables, we denote variables relative to the U.S. analog of that variable by the superscript $rel$.

The data is described in detail in Appendix C and comes from DataStream. The data covers 40 countries between 1982 and 2013. Not all countries have data for all years, resulting in an unbalanced panel. Furthermore, within the same country-year, some data is available while other data is not. For this reason, the number of data points (country-year) differs across different specifications, we report it as $n$ in all of the tables.

Table 1 presents results on the failure of UIP and the profitability of the carry trade (Panel A); the relationship between currency returns and output growth differentials, or the Balassa-Samuelson effect (Panel B); the relationship between currency returns and consumption growth differentials, or the Backus-Smith puzzle (Panel C); and the relationship between output growth and inflation (Panels D and E). We study these relationships either as a pooled regression across all countries and years, or as country-by-country regressions where we report the average correlation, average slope, fraction of slopes that are positive (negative in Panel D), and fraction of t-statistics that are significant. The results in this table are not new and are consistent with prior literature.

Panel A of Table 1 shows the well known failure of UIP and profitability of the carry trade. In this table, we present results of the following regression

$$\mathcal{Y}_{t+1} = a + b\left(r_t - r_t^{US}\right) + \epsilon_{t+1} \tag{1}$$

where $r_t$ and $r_t^{US}$ denote, respectively the interest rate in the foreign and in the domestic (U.S.) country.[9] In the row labeled "UIP", $\mathcal{Y}_{t+1} = -\Delta e = -\log(\frac{E_{t+1}}{E_t})$, i.e., the foreign currency appreciation against the U.S. dollar, and in the row labeled "CARRY", $\mathcal{Y}_{t+1} = r_{t+1}^C = \log(R_{t+1}^C) = -e_{t+1} + r_t - r_t^{US}$, i.e., the carry trade profit if the U.S. is the funding country.

If UIP holds, then $\Delta e_{t+1} = r_t - r_t^{US}$ implying that the slope coefficient in regression (1) with $\mathcal{Y}_{t+1} = -\Delta e_{t+1}$ is equal to $-1$. We find the slope to be insignificant from zero; point estimates are slightly negative in the pooled, but positive in the country-by-country approach. The failure of UIP suggests positive carry trade profits; indeed, we find carry trade profits to be positive and significant in both empirical approaches.[10]

Panel B of Table 1 presents the relationship between output growth differentials and contemporaneous currency appreciation. Specifically, we estimate the following regression

$$\mathcal{Y}_{t+1} = a + b\left(\Delta \log(GDP_t) - \Delta \log(GDP_t^{US})\right) + \epsilon_{t+1}. \tag{2}$$

The independent variable is the logarithm of real output growth relative to the U.S. The dependent variable $\mathcal{Y}_{t+1}$ is either the nominal appreciation of foreign currency, $-\Delta e_{t+1}$ or the real appreciation of foreign currency, $-\Delta er_{t+1}$. The real exchange rate $ER_t$ is defined as $ER_t = E_t \frac{P_t^{\mathrm{US}}}{P_t}$, where $P_t^{\mathrm{US}}$ and $P_t$ are, respectively, the U.S. and foreign price indices; note that we define a large $ER$ to mean a strong U.S. dollar. Countries with higher realized real output growth tend to contemporaneously experience both real and nominal currency appreciations. This is the Balassa-Samuelson effect (Balassa (1964), Samuelson (1964)).[11]

Corsetti, Dedola, and Leduc (2008) also find a similar positive relationship between realized consumption growth and real currency appreciations for the OECD. Similarly, Devereux and Hnatkovska (2014) find a positive relationship between realized consumption growth and real currency appreciations in international data, but a negative relationship in intra-national; because intra-national risk sharing is likely to be better than international, this suggests the importance of market incompleteness for international settings.[12]

[9] The interest rate is computed from forward rates using Covered Interest Parity: $r_t - r_t^{US} = -(f_t - e_t)$ where $f_t$ is the log of the forward rate.

[10] These results are consistent with many past studies. See Hodrick (2014) for a recent survey of the empirical literature on foreign exchange markets.

[11] See Tica and Druzic (2006) for a survey of empirical evidence on the Balassa-Samuelson effect.

[12] Devereux and Hnatkovska (2009) explain this in a model of incomplete risk sharing and both within and across country shocks.

As mentioned earlier, a positive relationship between currency appreciation and either output or consumption growth is inconsistent with the implications of most perfect risk sharing models (this is part of the Backus and Smith (1993) puzzle), although it is consistent with the model we develop in Section 3.[13] The Backus-Smith puzzle is evident in Panel C, which is identical to Panel B but uses consumption growth instead of output growth. Countries with higher realized consumption growth tend to contemporaneously experience both real and nominal currency appreciations.

Panels D and E of Table 1 document the negative relationship between real output growth and realized inflation. Supply shocks should lead to a negative relationship, while demand shocks to a positive relationship. As will be discussed in Section 4, our model implies that a negative relationship between output growth and inflation is one, of several, channels that can lead to positive carry trade profits; this is true even if risk sharing is perfect. Panel D uses the same cross-country data as panels A and B. The relationship between inflation and GDP growth is negative but only marginally significant, however it is negative and significant if we use inflation and GDP growth relative to the U.S. For our model, this second, relative relationship is more relevant. In Panel E, we test the same relationship but for a longer time series available for the U.S. only. We find this relationship to be negative but insignificant for GDP, however, negative and significant when we replace GDP by TFP.[14] This finding is consistent with the interpretation of TFP as a measure of supply shocks. Since GDP is a combination of demand and supply shocks, it is not surprising that we see a stronger relationship in TFP than in GDP.

In Table 2 we explore how country characteristics relate to interest rate differentials. Since carry trade target countries are, by definition, high interest rate differential countries, this table explores which country characteristics are driving the carry trade. In Panel A, we regress interest rate differentials at $t$ on several macroeconomic characteristics realized at $t+1$ or after. Even though the time $t$ variable is on the left hand side, these can be thought of as predictive regressions, asking what do interest rate differentials tell us about future macroeconomic performance. Surprisingly, the answer is not much. In contrast to the assumptions in several models, interest rate differentials are not explained by differences in future output growth $\Delta y^{rel}_{t+1}$, consumption growth $\Delta c^{rel}_{t+1}$, output

[13] Fundamentally, the Backus and Smith (1993) puzzle is only about consumption. However, in the data consumption and output growth have a correlation of 0.74 (it is also 0.74 is we instead use consumption and output growth relative to the U.S.). This correlation is also very high in most models of the carry trade. One could of course, construct models where this correlation is not high. For example in a model where all goods are tradable, and there are no frictions to trade, consumption growth will be perfectly correlated across all countries and there will be zero correlation between relative output and consumption growth rates.

[14] TFP data comes from John Fernald's website.

volatility $\sigma^{\Delta y,rel}_{t+1,t+4}$, or loading on world output growth $\beta^{\Delta y}_{t+1,t+4}$; although high interest countries do have higher consumption volatility $\sigma^{\Delta c,rel}_{t+1,t+4}$.[15] Interestingly, many models of the carry trade assume that the driving force behind the carry trade is a *lower* expected volatility of output or consumption growth in the high interest rate country – the results in this table are not supportive of such a channel.[16]

Differences in inflation expectations do appear related to interest rate differentials: high interest rate differential countries experience high inflation $i^{rel}_{t+1}$ and high inflation volatility $\sigma^{i,rel}_{t+1,t+4}$ going forward. The positive relation to future inflation is not surprising as the nominal interest rate, in part, reflects inflation expectations. Inflation is endogenous in our model, and depends on monetary policy. In our model, UIP holds with respect to expected inflation, thus while differences in expected inflation will lead to differences in nominal interest rates, they will not lead to carry trade profits. However, consistent with the data, our model implies that countries with higher inflation volatility do have higher interest rates. As will be shown below, consistent with the data, endogenous differences in inflation volatility are one driver of the carry trade and of violations of UIP in our model.

In Panel B of Table 2 we link interest rate differentials to three other contemporaneous country characteristics. High interest rate countries tend to be less open to financial trade (the measure of costs is from Garleanu, Panageas, and Yu (2015)). As will be shown below, consistent with the data, such costs of financial trade are a second driver of the carry trade and of violations of UIP in our model.

High interest rate countries are smaller, this was pointed out by Hassan (2013). This is typically inconsistent with habit models, such as Verdelhan (2010), where the target (high interest rate) country needs to be bigger.[17] On the other hand Hassan (2013) shows that shocks to non-tradable

[15]The results in this table are consistent with Gourio, Simer, and Verdelhan (2013)

[16]For example, the key driving force in Colacito (2009) and in Bansal and Shaliastovich (2012) is a higher volatility of non-tradable shocks in the low interest rate country; because there is perfect home bias, non-tradable output is equal to total output. The carry trade in Tran (2013) is profitable for exactly the same reason as in Bansal and Shaliastovich (2012), but the model allows for both tradables and non-tradables. The non-tradable sector needs to be either more volatile or more important for low interest rate countries. He provides some evidence that this is indeed true.

[17]This is because for this channel to work, the target country must have a lower risk aversion. A lower risk aversion is usually associated with bigger countries – positive shocks lead to larger size and to lower risk aversion as consumption is further from habit. An exception is Heyerdahl-Larsen (2014), where deep habit preferences allow the target country to be smaller.

output in a world with trade can lead to higher interest rates in smaller countries; we discuss this model in detail in Section 4.3.

High interest rate countries are also significantly more likely to default, as measured by their sovereign CDS spread.[18] This is not a purely mechanical relationship: in theory a country's interest rate may be high due to fundamentals (such as high expected consumption growth) affecting the risk free rate, or due to higher probability of default reflected in the CDS spread. As shown in Panel A of Table 2, fundamentals do not seem to have much of a relationship with interest rates; here we see that default risk does.[19] Interestingly, there is a significant and negative correlation (-0.12) between size and CDS spread, and once we control for CDS spread in multi-variate regressions (columns four and five), a country's size is no longer significantly related to its interest rate. As will be shown below, consistent with the data, the possibility of default is a third driver of the carry trade and of violations of UIP in our model.

Before moving on, it is relevant to mention another set of cross-country differences which may be responsible for the carry trade. Ready, Roussanov, and Ward (2016) show that carry trade target countries are commodity exporters. In their model, commodity exporters endogenously have a higher interest rate but are also safer because they are insulated from aggregate shocks resulting in positive carry trade profits. The model developed below has little to say about this channel.

In the last set of empirical results, we relate *realized* carry trade profits at $t+1$ to the interest rate differential - the typical signal for the carry trade - and additional country characteristics; these results are in Panel C of Table 2. The first row is always the coefficient on the interest rate differential (t-statistic in the second row), while the third row is the coefficient on an additional characteristic. For convenience, the first column presents the interest rate differential alone, with no controls; this coefficient is identical to the one in the CARRY row of Panel A, Table 1.

Realized carry trade returns are contemporaneously *positively* correlated with realized output and consumption growth in the target country relative to the U.S. This is, perhaps, the sharpest difference between our incomplete market model, and most of the alternative explanations of the carry trade mentioned above. Recall that, as in Backus and Smith (1993), complete markets typi-

[18]The CDS data comes from Datastream, it is described in Appendix C.

[19]Previously, Coudert and Mignon (2013) have shown that carry trades into high CDS countries do especially well during booms and especially poorly during busts, while Huang and MacDonald (2014) have shown that high interest rate currencies have a higher loading on aggregate sovereign credit risk.

cally imply that a currency depreciates after positive shocks, which results in a *negative* correlation. There is also a negative, but insignificant relation to realized inflation - our model implies that there should be a negative relationship.[20]

The relationships between realized carry trade profits, realized output shocks, and realized inflation are consistent with recent empirical work by Kim (2014) who investigates two specific carry trade strategies: borrowing in USD to invest in the relatively higher interest rate Australian Dollar (AUD), and borrowing in Japanese Yen (JPY) to invest in the relatively higher interest rate USD. For AUD, carry trade returns are highest when inflation and unemployment in the high interest rate country (Australia) are unexpectedly low. For JPY, carry trade returns are highest when machine orders in the low interest rate country (Japan) are unexpectedly low, and retail sales growth in the U.S. is unexpectedly high.

After controlling for interest rate differentials, realized carry trade profits at $t + 1$ do not seem related to country size or to financial openness at $t$. However, they are strongly related to the CDS spread at $t$. In fact, once the CDS spread is in the regression, the interest rate differential is no longer relevant for carry trade profit! Although these CDS result look strong, caution must be taken because the CDS data is available for a shorter time span.

The above results focus on realized output growth in the high interest rate (carry trade target) country, or relative output growth between the high and low interest rate countries. Most past studies have focused on the relationship between carry trade profits and aggregate world quantities. Gourio, Simer, and Verdelhan (2013) show that realized carry trade profits are higher when realized world output growth is high, that is the carry trade loads on aggregate risk; Lustig and Verdelhan (2007) suggest that the carry trade is risky because it is positively correlated with world output and consumption growth; Lustig, Roussanov, and Verdelhan (2011) show that the carry trade is more profitable when world volatility is high. These observations are also all true in our model, however, due to imperfect risk sharing, our model's more direct predictions are about country specific output.

[20] This result is also consistent with Bansal and Shaliastovich (2012), where counter-cyclical inflation is one of two channels that can lead to a carry trade. Unlike our model, in Bansal and Shaliastovich (2012), the process for inflation is specified exogenously.

# 3 Model

We consider a two-period general equilibrium model of a world economy consisting of two countries: Home ($H$) and Foreign ($F$). Each country produces two types of goods: Tradable ($T$) and Non Tradable ($N$). We assume that, within each country financial markets are complete and therefore allocations are determined in each economy by the optimal consumption and saving decisions of a representative household. The agents in each country receive a country-specific endowment of both tradable and non-tradable goods and can only trade in two nominal riskless bonds denominated in the domestic and foreign currency, respectively. This setup is standard, except for the market incompleteness (which resembles Corsetti, Dedola, and Leduc (2008) and Benigno and Thoenissen (2008)); later we also allow for default, which is non-standard. In what follows we describe the household optimization problem and derive the equilibrium conditions, Appendix B provides more details of the solution.

We denote by $C_{\mathrm{H,t}}$, $t = 0, 1$, the composite consumption good in H given by

$$C_{\mathrm{H,t}} = \left((1-\theta)(C^{\mathrm{T}}_{\mathrm{H,t}})^{\alpha} + \theta(C^{\mathrm{N}}_{\mathrm{H,t}})^{\alpha}\right)^{\frac{1}{\alpha}}, \quad 0 < \theta < 1, \tag{3}$$

where $C^{\mathrm{T}}_{\mathrm{H,t}}$ and $C^{\mathrm{N}}_{\mathrm{H,t}}$ denote, respectively, consumption of the T and N goods at time $t = 0, 1$. The parameter $\theta$ in (3) is the share of non-tradable goods in the consumption basket and $\alpha$ is the intratemporal elasticity of substitution between T and N goods.[21]

The agent receives an exogenous endowment of both tradable and non-tradable good at each date. Specifically, we denote by $Y^{\mathrm{T}}_{\mathrm{H},0}$ and $Y^{\mathrm{N}}_{\mathrm{H},0}$ the time-zero endowments of T and N goods and by $Z^{T}_{\mathrm{H}}Y^{\mathrm{T}}_{\mathrm{H},1}$ and $Z^{N}_{\mathrm{H}}Y^{\mathrm{N}}_{\mathrm{H},1}$ the time-1 endowments, with $Z^{T}_{\mathrm{H}}$ and $Z^{N}_{\mathrm{H}}$ representing random productivity shocks to tradables and non-tradables. The ratio $Y^{\mathrm{T}}_{\mathrm{H},1}/Y^{\mathrm{T}}_{\mathrm{H},0}$ and $Y^{\mathrm{N}}_{\mathrm{H},1}/Y^{\mathrm{N}}_{\mathrm{H},0}$ are the expected growth in the T and N endowment, respectively. If $Z^{T}_{\mathrm{H}}Y^{\mathrm{T}}_{\mathrm{H,t}} > C^{\mathrm{T}}_{\mathrm{H,t}}$ country H exports to country F at time $t = 0, 1$.

[21] Cobb-Douglas consumption aggregation is a special case of the aggregation (3), with $\alpha = 0$. More generally, $\alpha = -\infty$ implies $C_{\mathrm{H,t}} = \min(C^{\mathrm{T}}_{\mathrm{H,t}}, C^{\mathrm{N}}_{\mathrm{H,t}})$ or perfect complements; $\alpha = 0$ is Cobb-Douglas; $\alpha = 1$ implies $C = (1-\theta)C^{\mathrm{T}}_{\mathrm{H,t}} + \theta C^{\mathrm{N}}_{\mathrm{H,t}}$, or perfect substitutes; $\alpha \to \infty$ implies $C^{\mathrm{T}}_{\mathrm{H,t}} = \max(C^{\mathrm{T}}_{\mathrm{H,t}}, C^{\mathrm{N}}_{\mathrm{H,t}})$.

The household in the H country solves the following problem

$$\max_{\{C^{\mathrm{T}}_{\mathrm{H},0}, C^{\mathrm{N}}_{\mathrm{H},0}, B^{\mathrm{H}}_{\mathrm{H}}, B^{\mathrm{F}}_{\mathrm{H}}, C^{\mathrm{T}}_{\mathrm{H},1}, C^{\mathrm{N}}_{\mathrm{H},1}\}} \left( C_{\mathrm{H},0}^{1-\rho} + \beta \mathbb{E}\left[ C_{\mathrm{H},1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \quad 0 < \beta < 1, \tag{4}$$

with $\mathbb{E}$ denoting expectation conditional on information at time 0, subject to the following budget constraints at time 0 and 1

$$P^{\mathrm{T}}_{\mathrm{H},0} C^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0} C^{\mathrm{N}}_{\mathrm{H},0} + q_{\mathrm{H}} B^{\mathrm{H}}_{\mathrm{H}} + \frac{q_{\mathrm{F}}}{E_0} B^{\mathrm{F}}_{\mathrm{H}} = P^{\mathrm{T}}_{\mathrm{H},0} Y^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0} Y^{\mathrm{N}}_{\mathrm{H},0} \tag{5}$$

$$P^{\mathrm{T}}_{\mathrm{H},1} C^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1} C^{\mathrm{N}}_{\mathrm{H},1} = P^{\mathrm{T}}_{\mathrm{H},1} Z^{T}_{\mathrm{H}} Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1} Z^{N}_{\mathrm{H}} Y^{\mathrm{N}}_{\mathrm{H},1} + B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{E_1} B^{\mathrm{F}}_{\mathrm{H}} \tag{6}$$

where $P^{\mathrm{T}}_{\mathrm{H,t}}$ and $P^{\mathrm{N}}_{\mathrm{H,t}}$, $t = 0, 1$, denote the price in H currency of T and N goods; $B^{\mathrm{H}}_{\mathrm{H}}$ and $B^{\mathrm{F}}_{\mathrm{H}}$ denote the quantities of domestic and foreign bonds held by the household; $q_{\mathrm{H}}$ and $q_{\mathrm{F}}$ denote the nominal prices of the domestic and foreign bond; and $E_t$, $t = 0, 1$, denotes the exchange rate, expressed as the relative price of the foreign composite consumption good to the domestic composite consumption good. Note that a large $E$ implies a strong home currency.

In one version of the model, we add a cost of holding financing securities.[22] In this case, in the utility function (equation (4)), $C_{\mathrm{H},0}$ is replaced by $C_{\mathrm{H},0} - \kappa^{HH} \left( \frac{q_{\mathrm{H}} B^{\mathrm{H}}_{\mathrm{H}}}{P_{\mathrm{H},0}} \right)^2 - \kappa^{HF} \left( \frac{q_{\mathrm{F}} B^{\mathrm{F}}_{\mathrm{H}}}{E_0 P_{\mathrm{H},0}} \right)^2$ where the second and third terms are reduced-form utility costs of positions in domestic and foreign bonds, respectively. For example, this cost may represent information, monitoring, or creating collateralizable assets. We allow the cost $\kappa$ to be a function of both lender and borrower - this will later allow us to use the estimates of impediments to financial trade from Garleanu, Panageas, and Yu (2015) as an additional source of heterogeneity across countries. In most versions of the model $\kappa = 0$ and these costs are absent.

[22] Benigno and Thoenissen (2008) explore a similar cost, which helps to break risk sharing and explain the Backus-Smith puzzle. Alternately, Rabitsch (2014) assumes a fixed borrowing constraint, which, like a cost, limits financial trade and therefore risk sharing.

Consumption is determined by the the household's intra-temporal first order conditions combined with aggregate budget constraint:

$$\begin{aligned}
C^{\mathrm{T}}_{\mathrm{H},0} &= Y^{\mathrm{T}}_{\mathrm{H},0} - \frac{1}{P^{\mathrm{T}}_{\mathrm{H},0}}\left(q_{\mathrm{H}} B^{\mathrm{H}}_{\mathrm{H}} + \frac{q_{\mathrm{F}}}{E_0} B^{\mathrm{F}}_{\mathrm{H}}\right) && (7)\\
C^{\mathrm{T}}_{\mathrm{H},1} &= Z^{T}_{\mathrm{H}} Y^{\mathrm{T}}_{\mathrm{H},1} + \frac{1}{P^{\mathrm{T}}_{\mathrm{H},1}}\left(B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{E_1} B^{\mathrm{F}}_{\mathrm{H}}\right) && (8)\\
\frac{P^{\mathrm{N}}_{\mathrm{H},0}}{P^{\mathrm{T}}_{\mathrm{H},0}} &= \frac{\theta}{1-\theta}\left(\frac{C^{\mathrm{T}}_{\mathrm{H},0}}{C^{\mathrm{N}}_{\mathrm{H},0}}\right)^{1-\alpha} && (9)\\
\frac{P^{\mathrm{N}}_{\mathrm{H},1}}{P^{\mathrm{T}}_{\mathrm{H},1}} &= \frac{\theta}{1-\theta}\left(\frac{C^{\mathrm{T}}_{\mathrm{H},1}}{C^{\mathrm{N}}_{\mathrm{H},1}}\right)^{1-\alpha} && (10)
\end{aligned}$$

The market clearing condition for the N good implies

$$C^{\mathrm{N}}_{\mathrm{H},0} = Y^{\mathrm{N}}_{\mathrm{H},0} \quad \text{and} \quad C^{\mathrm{N}}_{\mathrm{H},1} = Z^{N}_{\mathrm{H}} Y^{\mathrm{N}}_{\mathrm{H},1}. \tag{11}$$

Bond holdings are determined by the household's inter-temporal first order condition (Euler equations)

$$q_{\mathrm{H}} = \mathbb{E}\left[\mathbb{M}_1 \frac{P_{\mathrm{H},0}}{P_{\mathrm{H},1}}\right] \quad \text{and} \quad q_{\mathrm{F}} = \mathbb{E}\left[\mathbb{M}_1 \frac{P_{\mathrm{H},0} E_0}{P_{\mathrm{H},1} E_1}\right], \tag{12}$$

where

$$\mathbb{M}_1 = \beta\left(\frac{C_{\mathrm{H},1}}{C_{\mathrm{H},0}}\right)^{-\rho}\left(\frac{C_{\mathrm{H},1}}{\mathbb{E}[C^{1-\gamma}_{\mathrm{H},1}]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma} \tag{13}$$

denotes the real stochastic discount factor. An equivalent set of intra- and inter-temporal first order condition holds for the household in the F country.

We define the aggregate H price level at time $t$ as

$$P_{\mathrm{H,t}} = \frac{P^{\mathrm{T}}_{\mathrm{H,t}} C^{\mathrm{T}}_{\mathrm{H,t}} + P^{\mathrm{N}}_{\mathrm{H,t}} C^{\mathrm{N}}_{\mathrm{H,t}}}{C_{\mathrm{H,t}}}. \tag{14}$$

Using the intra-temporal first order conditions, the above definition implies

$$P_{\mathrm{H,t}} = \left((1-\theta)^{\frac{1}{1-\alpha}} (P^{\mathrm{T}}_{\mathrm{H,t}})^{\frac{\alpha}{\alpha-1}} + \theta^{\frac{1}{1-\alpha}} (P^{\mathrm{N}}_{\mathrm{H,t}})^{\frac{\alpha}{\alpha-1}}\right)^{\frac{\alpha-1}{\alpha}}. \tag{15}$$

No arbitrage in the goods market implies that

$$P^{\mathrm{T}}_{\mathrm{H,t}} E_t = P^{\mathrm{T}}_{\mathrm{F,t}} \tag{16}$$

It is also possible to allow shipping costs, which we have done as an extension, however, for the sake of brevity, we focus on frictionless goods trade.[23]

The bond market clearing condition implies

$$B^{\mathrm{H}}_{\mathrm{F}} = -B^{\mathrm{H}}_{\mathrm{H}} \quad \text{and} \quad B^{\mathrm{F}}_{\mathrm{F}} = -B^{\mathrm{F}}_{\mathrm{H}}. \tag{18}$$

From the above equations, all quantities of interest are a function of eight unknowns: $q_{\mathrm{H}}$, $q_{\mathrm{F}}$, $B^{\mathrm{H}}_{\mathrm{H}}$, $B^{\mathrm{F}}_{\mathrm{H}}$, $P^{\mathrm{T}}_{\mathrm{H},0}$, $P^{\mathrm{T}}_{\mathrm{F},0}$, $P^{\mathrm{T}}_{\mathrm{H},1}$, $P^{\mathrm{T}}_{\mathrm{F},1}$. These unknowns will be determined by the household's four Euler equations, and by monetary policy. We define monetary policy as the Central Bank's choice of money supply $M_{\mathrm{H},0}$ and $M_{\mathrm{H},1}$, where $M_{\mathrm{H},1}$ is potentially a function of the realized shocks $(Z^T_{\mathrm{H}}, Z^T_{\mathrm{F}}, Z^N_{\mathrm{H}}, Z^N_{\mathrm{F}})$. Specifically, to close the model and determine the equilibrium price level, we assume that the amount of money $M_{\mathrm{H},0}$, $M_{\mathrm{H},1}$, $M_{\mathrm{F},0}$ and $M_{\mathrm{F},1}$ in the H and F country satisfy

$$M_{\mathrm{H},0} = P^{\mathrm{T}}_{\mathrm{H},0} Y^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0} Y^{\mathrm{N}}_{\mathrm{H},0} \tag{19}$$

$$M_{\mathrm{H},1} = P^{\mathrm{T}}_{\mathrm{H},1} Z^T_{\mathrm{H}} Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1} Z^N_{\mathrm{H}} Y^{\mathrm{N}}_{\mathrm{H},1} \tag{20}$$

Rearranging the above equations it is possible to write $P^{\mathrm{T}}_{\mathrm{H,t}}$ as a function of $M_{\mathrm{H,t}}$, $C^{\mathrm{T}}_{\mathrm{H,t}}$ (which itself is a function of $P^{\mathrm{T}}_{\mathrm{H,t}}$), and the four unknowns: $q_{\mathrm{H}}$, $q_{\mathrm{F}}$, $B^{\mathrm{H}}_{\mathrm{H}}$, $B^{\mathrm{F}}_{\mathrm{H}}$. Given an arbitrary monetary policy, the equilibrium is determined by the solution of remaining four equations (Euler equations) for the remaining four unknowns. We solve these four equations numerically by starting with a guess of the four unknowns, then updating the initial guess based on the errors of the four equations until the errors converge to zero.

We set $P^{\mathrm{T}}_{\mathrm{H},0} = P^{\mathrm{T}}_{\mathrm{F},0} = 1$, without loss of generality and consider two possible monetary policies:

[23] If one assumes "iceberg" shipping costs $\tau > 0$, then for every unit of home (foreign) good shipped abroad, only a fraction $1 - \tau$ arrives at destination. Obstfeld and Rogoff (2001) show that if markets are competitive, no arbitrage implies that

$$P^{\mathrm{T}}_{\mathrm{H,t}} = \begin{cases} (1-\tau) P^{\mathrm{T}}_{\mathrm{F,t}} \frac{1}{E_t}, & \text{if } Y^{\mathrm{T}}_{\mathrm{H,t}} > C^{\mathrm{T}}_{\mathrm{H,t}} \\ \frac{P^{\mathrm{T}}_{\mathrm{F,t}}}{1-\tau} \frac{1}{E_t} & \text{if } Y^{\mathrm{T}}_{\mathrm{H,t}} \leq C^{\mathrm{T}}_{\mathrm{H,t}} \end{cases}, \quad \text{for} \quad t = 0, 1. \tag{17}$$

1. Zero Expected Inflation (ZEI): $\mathbb{E}\left[P_{\mathrm{H},1}\right] = P_{\mathrm{H},0}$. The $t = 1$ money supply is a function of $t = 0$ variables only, i.e. it is a constant. This implies that the money supply at $t = 1$ is a function of $t = 0$ variables only. We believe this case to be relevant empirically because if the central bank cannot observe economic shocks in real time, but if low inflation is desired (for example due to menu costs), then the central bank can only target zero expected inflation; furthermore, empirically, inflation volatility is not zero.

2. Zero Realized Inflation (ZI): $P_{\mathrm{H},1} = P_{\mathrm{H},0}$. The $t = 1$ money supply is potentially a function of $t = 1$ variables.

### 3.1 Calibration

We now describe the parameter choices under which we solve the model. We set $\theta = 0.5$, which implies that the fraction of non-tradables in consumption is 50%.[24] We set $\alpha = 0$, which implies Cobb-Douglas aggregator over tradable and non-tradable consumption.[25] We set the intertemporal elasticity of substitution, $\frac{1}{\rho} = 1.5$ and the risk aversion, $\gamma = 7.5$. This is a popular calibration of the long run risk model, used in Bansal and Yaron (2004). When there is asymmetry in risk aversion, we set $\gamma^F = 10$ in F and $\gamma^H = 5$ in H.

Both countries face equal sized, independent productivity shocks ($Z_{\mathrm{H}} = Z_{\mathrm{F}} = \{0.93, 1.0, 1.07\}$) with equal probability in each of the nine states of the word. In some versions of the model, shocks are in the tradable sector only, in others in the non-tradable sector only,[26] implying real GDP volatility of around 3%. When there is asymmetry in volatility, the shocks in F are twice as volatile as in H ($Z_{\mathrm{F}} = \{0.9067, 1.0, 1.0933\}$ and $Z_{\mathrm{H}} = \{0.9534, 1.0, 1.9466\}$) implying real GDP volatility of around 2% in H and 4% in F. The expected size of each country's tradable and non-tradable output is one: $Y^{\mathrm{T}}_{\mathrm{H,t}} = Y^{\mathrm{T}}_{\mathrm{F,t}} = Y^{\mathrm{N}}_{\mathrm{H,t}} = Y^{\mathrm{N}}_{\mathrm{F,t}} = 1$, when there is asymmetry in size, we double the size of F.

Some of the models have costs of financial trade $\kappa$. In those models when costs are non-zero, either $\kappa = 0.01$ for all investments; or $\kappa^{HF} = 0.01$ when H invests into F, and zero for all other

[24] Corsetti, Dedola, and Leduc (2008) estimate that 53% of consumption in the U.S. is non-traded; Stockman and Tesar (1995) find a value of 50% for OECD countries.

[25] Stockman and Tesar (1995) estimate $\alpha = -1.27$ for a sample of developed and developing countries; Mendoza (1991) estimates $\alpha = -0.35$ for industrialized countries; Obstfeld and Rogoff (2001) set $\alpha = 5/6$. We have experimented with alternative values of $\alpha$ and the results are generally similar to those with $\alpha = 0$.

[26] The model is capable of having shocks to both sectors, but we do not present such cases.

investments. Some of the models allow for default, in those models, default occurs only in country F and only in the state of the world where both countries receive the worst possible shock. Conditional on defaulting, F repays only 75% of the nominal value of its bonds. Section 4.4 provides further discussion of these parameters.

# 4 Analysis

In Sections 4.1, 4.2, and 4.3 we present models with asymmetries in output volatility, risk aversion, and size across countries. Although we do not believe these asymmetries to be the most relevant empirically, it is useful to study them because they develop intuition about what incomplete markets can and cannot do, and because these are the asymmetries that the academic literature has so far focused on. Based on our empirical findings in Section 2, we believe the most relevant asymmetries are in costs of financial trade, and in sovereign default risk. These are presented in Section 4.4.

## 4.1 Asymmetry in volatility

One common way to model the carry trade is through an asymmetry in volatility across countries. In this section we will analyze four models all having asymmetry in output volatility.

1. The first model has complete financial markets and asymmetry in the volatility of non-tradable shocks, this is the channel in Colacito (2009), Bansal and Shaliastovich (2012), and Tran (2013).[27] This model delivers positive carry trade profit, however, in contrast to the data, currency returns are counter-cyclical.

2. The second model also has complete financial markets but the asymmetry is in tradable shocks rather than non-tradable shocks, which, as mentioned in Section 2, is more relevant empirically. However, tradable shocks alone cannot produce interesting exchange rate and carry trade behavior: in this model the exchange rate is constant and the carry trade profit is zero.

3. The third model has asymmetry in tradable shocks, like the second, but financial markets are limited to nominal bonds and the government follows a zero expected inflation monetary

[27] Tran (2013) allows for both tradable and non-tradable goods, but the non-tradable shocks are relatively more important, as a result the driving force behind the carry trade is very similar to Bansal and Shaliastovich (2012).

policy. The real side of this economy is identical to the second model, but, due purely to nominal forces, this model has positive carry trade profits and a pro-cyclical nominal currency return - as in the data.

4. The fourth model is an extension of the second but with costs of financial trade; this model has an additional real channel for the carry trade profit and for pro-cyclical currency return, which reinforces the nominal channel.

#### 4.1.1 Complete markets and non-tradable shocks

In the first model, there is a full set of Arrow-Debreu securities. We shut down the tradable shock and allow for non-tradable shocks only; this shock is twice as volatile in F as in H. Though the model is not quite identical to Bansal and Shaliastovich (2012), the driving force behind currency returns and carry trade profits is exactly the same.[28] Appendix A explains the mechanics and intuition of such models. The results of this model are in the first row of Table 3, Panel A.

As in Bansal and Shaliastovich (2012), this model implies that due to precautionary savings, the high volatility country (F) has a lower interest rate. The carry trade of borrowing in F and investing in H is risky and therefore profitable on average. However, this model has important features at odds with the data. First, the correlation of the target (high interest rate) country's currency appreciation (and therefore the carry trade profit, since there is no default) with the target country's output is negative - in the data it is positive. Second, it relies on shocks to non-tradables - in the data tradable shocks seem to be more relevant. Third, the target country has a lower volatility of output than the funding (low interest rate) country - in the data there is no difference in volatility.

#### 4.1.2 Complete markets and tradable shocks

We will now tweak this model in several ways to bring it closer to the data. In the second row of Panel A, we switch from non-tradable shocks only, to tradable shocks only. Unlike non-tradable shocks, tradable shocks are easy to redistribute through trade, and result in the model becoming

[28] The differences are (i) Bansal and Shaliastovich (2012) is a dynamic model whereas this model has two periods only, (ii) Bansal and Shaliastovich (2012) have no goods trade – which is equivalent to having non-tradable goods only, or $\theta = 1$ in our framework – whereas our model allows for trade in tradable goods. However, it is unimportant because the shock is to non-tradables.

very boring: consumption is perfectly correlated across countries, the nominal and real exchange rates are constant, and the carry trade profit is exactly zero. Thus, the nature of the shocks matters: if the crucial shocks are to the tradable sector and if markets are complete, then some of the conclusions reached by models with no trade may be invalid. This observation is empirically relevant as well: estimates of shocks across sectors suggest that tradable shocks are significantly more volatile than non-tradable shocks.

#### 4.1.3 Incomplete markets and tradable shocks

Next, we abandon complete markets and allow for trade in nominal bonds only; we also assume that the monetary authority follows a zero expected inflation policy.[29] This model is in the third row of Table 3, Panel A. The zero expected inflation monetary policy leads to counter-cyclical inflation. This is because after a positive output shock to F, prices in F fall because monetary policy is not state dependent and goods are relatively more abundant. Counter-cyclical inflation is important for some of the results we discuss below, and as shown in Section 2, it is also consistent with the data.[30]

Interestingly, nominal bonds in this environment still allow for perfect risk sharing; this is evident from comparing the real quantities in rows two (complete markets) and three (nominal bonds only) of Panel A – they are identical. The reason is that counter-cyclical inflation implies that the real payoff of nominal bonds is pro-cyclical. Thus, after a positive shock in F, H investors in F-bonds can purchase more tradable goods and ship them back to Home; an alternative way of saying this is that the currency of F appreciates ($E = \frac{P_F^T}{P_H^T}$ falls) making H-investors better off. Thus, F-bonds behave like F-equity and are a perfect risk sharing security.

Despite the real quantities being identical, this model has interesting implications for the carry trade. Unlike Bansal and Shaliastovich (2012), it is now the low volatility country H that has lower interest rates. The carry trade of borrowing in H and investing in F is profitable, this is due to

[29]We are not able to obtain a solution when shocks are to tradables only and the monetary authority follows a zero realized inflation policy. In Appendix B.3, we prove that no solution exists for the case tradable shocks, zero realized inflation, logarithmic utility, and symmetric shocks. We cannot prove this for the general case, but our numerical results suggest that the general case also has no solution.

[30]Counter-cyclical inflation usually arises from supply shocks as higher output leads to lower prices. On the other hand, demand shocks combined with some form of rigidity could lead to pro-cyclical inflation as higher demand induces both higher output and higher prices. An unexpected shock to the money supply and sticky prices may also lead to higher prices and higher output. In the real world demand, supply, and monetary shocks could all be important, however, in Table 2 we document that empirically, inflation is indeed counter-cyclical.

a purely monetary channel since the real side of this economy is identical to the boring complete markets case. This can also be explained by counter-cyclical inflation. Though inflation is counter-cyclical (with respect to local shocks) in all countries, countries with more volatile shocks have a higher inflation volatility. As will be explained below, this higher inflation volatility is responsible for the higher interest rate in the higher volatility country, and for the positive carry trade profit; this is also consistent with the data, where high interest countries have higher inflation volatility.

Counter-cyclical inflation volatility implies pro-cyclical (relative to local shocks) currency returns because $E_1 = \frac{P^{\mathrm{T}}_{\mathrm{F},1}}{P^{\mathrm{T}}_{\mathrm{H},1}}$ e.g., a positive shock to F leads to a fall of tradable prices in F, a drop in $E_1$, and hence an appreciation of F currency. Furthermore, since shocks to the more volatile country (F) matter more for aggregate world consumption, F's currency is more pro-cyclical with respect to world shocks as well. Therefore holding F's currency is riskier and the trade of borrowing in H to invest in F must earn a premium.

For the trade above to be defined as the carry trade, it must also be the case that F has a higher nominal interest rate. This is the case because F's inflation is more volatile, making the real payoffs of its nominal bonds more pro-cyclical with respect to aggregate world shocks. To see this mathematically, note that the nominal bond price is:

$$q = \mathbb{E}\left[\mathbb{M}_{t+1}\frac{P_t}{P_{t+1}}\right] = corr\left[\mathbb{M}_{t+1}, \frac{P_t}{P_{t+1}}\right]\sigma\left[\frac{P_t}{P_{t+1}}\right]\sigma\left[\mathbb{M}_{t+1}\right] + \mathbb{E}[\mathbb{M}_{t+1}]\mathbb{E}\left[\frac{P_t}{P_{t+1}}\right]. \tag{21}$$

Since the real stochastic discount factor $\mathbb{M}_{t+1}$ is identical for both countries due to perfect risk sharing, the nominal interest rate can be high ($q$ is low) if $corr\left[\mathbb{M}_{t+1}, \frac{P_t}{P_{t+1}}\right]\sigma\left[\frac{P_t}{P_{t+1}}\right]$ is large in magnitude (more negative)[31], or $\mathbb{E}[\frac{P_t}{P_{t+1}}]$ is small. The country with a higher loading on aggregate risk has both a more negative $corr\left[M_{t+1}, \frac{P_t}{P_{t+1}}\right]$, and a more volatile inflation (high $\sigma\left[\frac{P_{t+1}}{P_t}\right]$ and high $\sigma\left[\frac{P_t}{P_{t+1}}\right]$); this pushes interest rates up. At the same time, due to Jensen's inequality, the country with a higher loading on aggregate risk has a higher $\mathbb{E}[\frac{P_t}{P_{t+1}}]$, pushing interest rates down; in most cases, this Jensen's inequality effect is small and the high aggregate risk country has a higher nominal interest rate.[32]

[31] Note that $corr[M_{t+1}, \frac{P_t}{P_{t+1}}] < 0$. This is because $\mathbb{M}_{t+1}$ is counter-cyclical, as is inflation ($\frac{P_{t+1}}{P_t}$).

[32] Because of convexity, Jensen's inequality implies $\mathbb{E}\left[\frac{P_t}{P_{t+1}}\right] > \frac{\mathbb{E}[P_t]}{\mathbb{E}[P_{t+1}]} = 1$. The only time the Jensen's inequality term matters is when the volatility of F-shocks is much higher than of the H-shock, and the risk aversion is sufficiently low. In this case, $\sigma[M_{t+1}]$ is very small and the term $\mathbb{E}[\frac{P_t}{P_{t+1}}]$ dominates, resulting in a lower nominal interest rate in the high aggregate risk country.

### 4.1.4 Incomplete markets, tradable shocks, and financial trade costs

Finally, in the fourth row of Table 3, Panel A, we restrict financial trade further so that risk sharing is no longer perfect. To do so, we introduce a cost $\kappa$ of buying financial assets.

Prior to discussing this case, it is useful to review the intuition behind the Balassa-Saumelson effect, that is the observation that real currency appreciation is positively correlated with output shocks. The basic intuition can be seen in a static, one period model, however, this easily extends to a dynamic model in which lack of financial markets prevents agents from trading intertemporally.[33] Since there is only one tradable good, there can be no mutual gains from trade, therefore each country just consumes its endowment. Consider a monetary policy setting the price of tradables to one,[34] a positive shock to tradable output must therefore increase the price of non-tradables, as they are relatively less abundant. This leads to a rise in the overall price level, and a real currency appreciation. More generally, even if the price of tradables falls (which is required for a nominal currency appreciation), as long as the aggregate price level falls by less, the real currency will appreciate.[35]

When $\kappa > 0$, the model gets closer to this simple example of the Balassa-Saumelson effect. Due to costs, there are fewer financial claims traded – in this parametrization, the total value of financial securities traded is one third of the perfect risk sharing case. This leads to less intertemporal trade, and to each country's consumption more closely resembling its output. Risk sharing is no longer perfect and consumption growth is no longer perfectly correlated across countries. A positive shock to tradable output in H leads to a real appreciation of H's currency due to the Balassa-Samuelson channel. The carry trade is now risky because the real exchange rate is now volatile and pro-cyclical, therefore the F currency will depreciate by even more when F suffers negative shocks. Note that the nominal and real channels work in the same direction and reinforce each other because both make currency pro-cyclical.

[33]In such a model, there is no trade because there is no way for one country to promise its future output in return for another country's excess output today.

[34]This assumption is innocuous. Since consumption is pre-determined by endowment, monetary policy cannot have an effect on real quantities, including the real exchange rate.

[35]Recall that $ER_t = E_t \frac{P_{\mathrm{H,t}}}{P_{\mathrm{F,t}}} = \frac{P^{\mathrm{T}}_{\mathrm{F,t}}}{P^{\mathrm{T}}_{\mathrm{H,t}}} \frac{P_{\mathrm{H,t}}}{P_{\mathrm{F,t}}}$ thus the real currency of F appreciates ($ER_t$ falls) if $P_{\mathrm{F,t}}$ rises by more than $P^{\mathrm{T}}_{\mathrm{F,t}}$ rises, or $P_{\mathrm{F,t}}$ falls by less than $P^{\mathrm{T}}_{\mathrm{F,t}}$ falls. In other words, if the nominal currency appreciation is zero, then the real currency appreciates if the price level rises; if the nominal currency appreciates, then the real currency appreciates if the price level falls by more than the price of tradables falls.

To sum up the results of this section, incomplete markets combined with asymmetric volatility 1) lead to positive carry trade profits, as in the data; 2) lead to a positive correlation between carry trade profit into the high interest rate country, and currency appreciation, output growth, and consumption growth of the same country, as in the data; 3) imply a positive relationship between interest rate differentials and inflation volatility, as in the data; 4) the above points rely on shocks to tradables, which are more important than non-tradables in the data. These points are in contrast to existing models. However, these results require the high interest rate country to have more volatile output; existing models require the high interest rate country to have less volatile output, but there seems to be no link between output volatility and interest rate differential in the data.

### 4.2 Asymmetry in risk aversion

As an alternative to asymmetry in volatility, several explanations of the carry trade focus on asymmetry in risk aversion, which usually comes about due to a history of past shocks in a world with habit utility. We conduct the same four experiments as in the previous section, but with this asymmetry.

1. The first model has complete financial markets and non-tradable shocks only, this is the channel in Verdelhan (2010) and Heyerdahl-Larsen (2014). Like the complete market models in the previous section, this model delivers positive carry trade profit, however, in contrast to the data, currency returns are counter-cyclical.

2. The second model also has complete financial markets but tradable instead of non-tradable shocks; switching to tradable shocks reduces the profitability of the carry trade, although not completely as in the previous section.

3. The third model has tradable shocks, like the second, but financial markets are limited to nominal bonds and the government follows a zero expected inflation monetary policy. Nominal currency returns are now pro-cyclical - as in the data, but UIP holds and the carry trade profit is zero.

4. The fourth model is an extension of the third but with costs of financial trade; in this model real and nominal currency appreciation is pro-cyclical and the carry trade is profitable.

### 4.2.1 Complete markets and non-tradable shocks

In the first model, there is a full set of Arrow-Debreu securities, non-tradable shocks only, and the risk aversion in F is twice that in H. Though not quite identical to Verdelhan (2010), the driving force behind currency returns and carry trade profits is exactly the same.[36] Appendix A explains the mechanics and intuition of such models. The results of this model are in the first row of Table 3, Panel B.

As in Verdelhan (2010), this model implies that due to precautionary savings, the high risk aversion country (F) has a lower interest rate. The carry trade of borrowing in F and investing in H is risky and therefore profitable on average. However, this model has important features at odds with the data. First, the correlation of the target (high interest rate) country's currency appreciation (and therefore the carry trade profit, since there is no default) with the target country's output is negative - in the data it is positive. Second, it relies on shocks to non-tradables - in the data tradable shocks seem to be more relevant. Third, the target (high interest rate) country is bigger than the funding (low interest rate) country[37] - in the data target countries are smaller.

### 4.2.2 Complete markets and tradable shocks

We again tweak this model to bring it closer to the data. In the second row of Panel A, we switch from non-tradable shocks only, to tradable shocks only. Unlike non-tradable shocks, tradable shocks are easy to redistribute through trade, which reduces the volatility of the exchange rate, the interest rate differential, and the carry trade profit. Unlike the previous section, where the asymmetry was due to volatility, these quantities do not go all the way to zero. However, the take away is the same as before, the nature of the shocks matters; recall that tradable shocks tend to be more important than non-tradable shocks in the data.

[36] The differences are (i) Verdelhan (2010) is a dynamic model where differences in risk aversion arise due to the interaction of past shocks and habit preferences, whereas this model has two periods only and exogenously specified risk aversion, (ii) Verdelhan (2010) has no goods trade – which is equivalent to having non-tradable goods only, or $\theta = 1$ in our framework – whereas this model allows for trade in tradable goods. However, it is unimportant because the shock is to non-tradables.

[37] Though in our two model example, the two countries are of the same size, in a dynamic habit model, the high risk aversion (low interest rate) country has high risk aversion precisely because it has received negative shocks in the past and is now smaller.

#### 4.2.3 Incomplete markets and tradable shocks

Again, we abandon complete markets and allow for trade in nominal bonds only; we also assume that the monetary authority follows a zero expected inflation policy. This model is in the third row of Table 3, Panel B. Recall that this monetary policy leads to counter-cyclical inflation and pro-cyclical real returns on nominal bonds, which makes these bonds behave like equity. The correlation of consumption growth is perfect, and the ratio of consumption growth volatility is is one, which would be optimal if the two countries had the same utility. However, with F being more risk averse, optimal risk sharing implies perfect correlation but lower consumption growth volatility for F, as in row two. Thus, unlike the case of asymmetry in volatility, nominal bonds are no longer enough to achieve perfect risk sharing. That is because in the previous section, the optimal policy was for each country to buy the other's equity in equal amounts, which was achievable through trade in equal quantities of nominal bonds. In the case of asymmetry in risk aversion, the optimal policy is for the low risk aversion country to borrow from the high risk aversion country and hold a relatively larger equity position so that it can have a higher consumption growth volatility. That is not possible when the only assets are two equity-like nominal bonds. Instead, an equal size exchange in the two bonds results in the two countries having perfectly correlated consumption growth with the same volatility - this would be optimal if the two had the same risk aversion, but is suboptimal since F is more risk averse.

Since F is risk averse, its implied real interest rate (which is not a traded return) is lower than in H, however its nominal interest rate is higher. This again has to do with the equity-like nature of nominal bonds. Because F is more risk averse, its equity-like nominal bond is perceived as very risky. This can be seen mathematically in equation (21) where the first term on the right is more negative in F.[38] UIP holds in this economy, thus, despite a higher nominal interest rate, the carry trade profit is zero.

[38] Inflation and the stochastic discount factor are both counter-cyclical, however their covariance is higher in F. This implies that the first term in equation (21) is more negative in F. The first term consists of the correlation of the stochastic discount factor and deflation, which is positive and similar in both countries; and the stochastic discount factor, which is bigger in F.

#### 4.2.4 Incomplete markets, tradable shocks, and financial trade costs

Finally, as in the last section, in the fourth row of Table 3, Panel B, we restrict financial trade further so that risk sharing is no longer perfect. To do so, we introduce a cost $\kappa$ of buying financial assets. Due to even more limited financial trade (again roughly one third of the amount in the costless case), risk sharing is even worse: the volatilities of consumption growth are still identical (optimally F would have a lower volatility), but now their correlation is below one. This leads to an even higher nominal interest rate in F, because F's stochastic discount factor is even more volatile - the intuition is the same as in the costless case. Additionally, due to the Balassa-Samuelson intuition described in Section 4.1, the real exchange rate is now pro-cyclical and the carry trade is profitable.

The results in this section parallel those of Section 4.1: incomplete markets combined with asymmetric risk aversion 1) lead to positive carry trade profits, as in the data; 2) lead to a positive correlation between carry trade profit into the high interest rate country, and currency appreciation, output growth, and consumption growth of the same country, as in the data; 3) the above points rely on shocks to tradables, which are more important than non-tradables in the data. These points are in contrast to existing models.[39]

### 4.3 Asymmetry in size

Hassan (2013) provides an alternative explanation of the carry trade, which relies on differences in size across countries. As in the models described earlier, non-tradable shocks are crucial – the intuition is described below. An important contribution of this modeling framework is that it allows small countries to have higher interest rates and to be carry trade targets - size is one of the few characteristics related to interest rate differentials in the data.

We conduct the same four experiments as in the previous section, but with asymmetry in size.

1. The first model has complete financial markets and non-tradable shocks only, this is the basic channel in Hassan (2013); this model delivers positive carry trade profits and higher

[39] Though this is outside the model, these results also suggest a possible explanation for why smaller countries tend to have higher interest rates: if, as in Verdelhan (2010), smaller countries are smaller due to past negative shocks, which, in combination with habit utility, also leads them to be more risk averse, then smaller countries will have higher interest rates and the carry trade into these countries will be profitable.

interest rates in smaller countries, however in contrast to the data, currency returns are counter-cyclical.[40]

2. The second model also has complete financial markets but tradable instead of non-tradable shocks; as the second model in Section 4.1, this model is boring in that it has a constant exchange rate and zero carry trade profit.

3. The third model has tradable shocks, like the second, but financial markets are limited to nominal bonds and the government follows a zero expected inflation monetary policy. As in Section 4.1, the real side of this economy is identical to the second model, but, due purely to nominal forces, this model has positive carry trade profits and a pro-cyclical nominal currency return - as in the data. However, it is now the large country that has a high interest rate.

4. The fourth model is an extension of the second but with costs of financial trade; this model has an additional real channel for the carry trade profit and for pro-cyclical currency return, which reinforces the nominal channel.

### 4.3.1 Complete markets and non-tradable shocks

In the first model, there is a full set of Arrow-Debreu securities, non-tradable shocks only, and the size of F is twice that in H. Though not quite identical to Hassan (2013), the driving force behind currency returns and carry trade profits is exactly the same. The results of this model are in the first row of Table 3, Panel C. As in Hassan (2013), the smaller country (F) has a higher interest rate and the carry trade investing in that country is profitable. However, F's currency return is negatively related to F's output growth.

The intuition is that after a negative shock to a country, due to risk sharing, the country consumes more tradable goods shipped from the rest of the world. A bond in that country pays one unit of its consumption, which, denominated in terms of tradables, will be very valuable since this country consumes relatively many tradables and the price of its non-tradables is very high. Thus, the bond of any country is a good hedge against its non-tradable shocks. Due to risk sharing across the world, non-tradable shocks to a large country are riskier than to a small country from the

[40] An extension in Hassan (2013) allows for an exogenously specified inflation process which also affects currency returns. Because currency returns are high when inflation is low, it is possible for currency returns to be positively correlated with output if inflation is specified to be negatively correlated with output.

point of view of the rest of the world, therefore bonds of large countries will be relatively expensive and carry lower interest rates. Bonds of smaller countries are worse hedges, carry higher interest rates, and require a risk premium to hold. However, at the same time, due to the complete market intuition of Backus and Smith (1993), any country's currency appreciates during bad times.

#### 4.3.2 Complete markets and tradable shocks

The same model but with tradable shocks is in the second row. As in Section 4.1, this model is boring: consumption is perfectly correlated across countries, the nominal and real exchange rates are constant, and the carry trade profit is exactly zero. As before, this is because tradable shocks are easy to redistribute through trade. Although the model with non-tradable shocks fits the data well when considering differences in size, the relatively bigger importance of tradable shocks in the data, and the correlation between currency appreciation and output growth in the data, suggest that a richer model, perhaps with important roles for tradable and non-tradable shocks, is required. Another possibility is that size is just correlated with another characteristic which drives the carry trade – our empirical results in Section 2 suggest that sovereign default risk may be one such characteristic and we explore it in the next section.

#### 4.3.3 Incomplete markets

Analogous to previous sections, in row three of 3, Panel C we present an incomplete market model with trade in nominal bonds only and with a zero expected inflation policy. An extension of this model with a cost of financial trade is in the fourth row. The results and intuition are exactly analogous to Section 4.1 with 'larger country' replacing 'more volatile country' - the intuition is the same because these characteristics make a country more important for world output shocks. Thus, in the third row, the carry trade is profitable for purely nominal reasons, and in the fourth row, it is profitable for both real and nominal reasons. However, unlike the data, this model implies that larger countries have higher interest rates.

## 4.4 Alternative asymmetries

In this section we explore two alternative channels for cross-country differences that may drive the carry trade, which are motivated by our empirical results. The first is cross-country differences in default risk. Several other studies have tied carry trade profits to downside risk and the peso problem, these include Burnside (2011), Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011), and Lettau, Maggiori, and Weber (2014); default, either actual or through inflation, would be one way in which this downside risk could arise. The second is cross-country differences in impediments or costs of financial trade. Garleanu, Panageas, and Yu (2015) show that certain countries are systematically more difficult to invest in from abroad; for example this may be a cost of monitoring or acquiring information.

### 4.4.1 Asymmetry in sovereign default

We assume that the bonds of F may suffer a nominal default in some states of the world. This may be because the financial market in F is not sophisticated enough to commit to always pay back its promises. In particular, F only repays 75% of the nominal value of its debt in any default state. For simplicity, we assume default happens in only one state of the world – the state in which both countries suffer the most negative realization of the shock, we call this a *systemic default* – and the nominal bond is fully repaid in all others.[41]

We solve this model for a case of zero inflation (thus nominal and real values are identical), and a case of zero expected inflation, these are presented in Panel D of Table 3.[42] The above assumption about default implies an interest rate differential of roughly 3% between F and H in our model.[43] In the data, when countries are sorted on interest rate differential versus the U.S., the lowest quintile is around $-2\%$ while the highest is above 7%. The mean CDS in our data is 0.89%,

[41] In our baseline model, there are three possible states for each country's tradable output: low, medium, and high. In total, there are nine equally likely states and default occurs in low/low. As an alternative, we define a *strategic default* as an event that occurs when H suffers the most negative realization of the shock, while F has the most positive realization. If there was no default, this would be the state of the world in which the most goods are shipped from F to H, because H redeems its highly valuable (in real terms) F-bond. We call this strategic default because this is the state of the world in which F is most incentivized to default. We believe systematic default is most relevant empirically and examine these cases in the main text. We have also solved cases of strategic default, those results are mostly similar but quantitatively weaker.

[42] As an alternative, we have also solved cases where the nominal value is repaid in full, however the money supply is increased in one state of the world, leading to high inflation and a smaller payout in real terms. For the sake of brevity, we do not present these results, though they are similar to the cases of nominal default.

[43] The probability of default is $1/9$ and the loss given default is 25% implying an approximate spread of $1/9 \times 0.25$.

with a standard deviation of 2.4%. These numbers suggest that a 3% spread between F and H is reasonable.

Recall that, from the previous analysis, the reason that two nominal bonds were often enough for perfect risk sharing is that their real returns were perfectly inversely correlated with the price level, making them behave like equity. Now that the nominal payoff of one of the bonds is no longer constant, these two bonds are no longer enough for perfect risk sharing and, as a result, consumption growth correlation is below one. These cases are analogous to $\kappa > 0$ in that far fewer bonds are issued, however instead of a trading cost, here the reason for less financial trade is that these bonds are an imperfect risk sharing tool.[44]

The consumption growth correlation between the two countries is now only 0.11 with zero inflation and 0.36 with zero expected inflation – these numbers are similar to the data. Despite having identical output volatility, the volatility of Foreign consumption is lower than Home – on this dimension, the model does not fit the data. This happens because when F defaults and H does not, then F is, in effect, getting a transfer from H. Fixing this remains a direction for future work. One possible solution is to extend the model from two periods to a fully dynamic model; for example if F is excluded from financial markets because of potential future default or as punishment for past default, as in Eaton and Gersovitz (1981), then F's consumption may become more volatile. Another possibility is to more explicitly link sovereign default to the rest of the economy, for example Xu (2015) shows that sovereign default is associated with runs on private banks and additional costs for the economy.

Since default happens in the worst aggregate state (low/low), F has relatively high consumption in the worst state while H has relatively low consumption. To compensate H investors for low consumption in the default state, F-bonds pay a higher nominal rate and H investors have higher consumption in non-default states. This is also why the carry trade is risky: F-bonds lose value exactly when H investors want insurance most.

Real domestic currency appreciation is now quite volatile and positively related to domestic output shocks. This is the Balassa-Samuelson effect and it is present in all models with default risk. As mentioned above, far fewer bonds are issued and far less risk sharing occurs. As a result, a positive domestic output shock leads to an abundance of tradables not only in production but also

[44]The nominal value of outstanding bonds relative to the perfect risk sharing case is roughly 28%.

in consumption. Although the price level falls after a positive shock (which would normally lead to a real currency depreciation), the price of tradables falls by relatively more because, due to the relative scarcity of non-tradables, the price of non-tradables relative to tradables must rise. This causes the nominal and real currency to appreciate ($ER = E_1 \frac{P_{\mathrm{H},1}}{P_{\mathrm{F},1}} = \frac{P^{\mathrm{T}}_{\mathrm{F},1}}{P^{\mathrm{T}}_{\mathrm{H},1}} \frac{P_{\mathrm{H},1}}{P_{\mathrm{F},1}}$).

Real domestic currency appreciation is also positively related to the difference between domestic and foreign consumption growth (not in table). This is because risk sharing is no longer perfect and there is a positive relationship between output growth differentials and consumption growth differentials; this would not be the case if risk sharing was perfect. As mentioned earlier, the Backus-Smith puzzle is that under perfect risks sharing, real domestic currency appreciation is negatively related to the difference between domestic and foreign consumption growth despite a positive relationship in the data. Thus, our model resolves the Backus-Smith puzzle; Benigno and Thoenissen (2008) and Corsetti, Dedola, and Leduc (2008) have found similar results in other settings where risk sharing is imperfect.

Comparing the purely real model to the model with inflation, we see that the latter allows for better risk sharing and the carry trade is weaker. Inflation has two effects. Recall that because of the zero expected inflation monetary policy, inflation is counter-cyclical. It is highest in the worst state of the world, the state where F defaults. In equilibrium, F invests in H-bonds and H invests in F-bonds; when F defaults, it still redeems all of its investments in H, but does not pay to H the full face value of the F-bond. Because this shortfall is denominated in nominal terms, high inflation in the default state makes the real value of the shortfall smaller, compared to a purely real model. As a result, risk sharing is better in the model with counter-cyclical inflation, which leads to a smoother real exchange rate and a smaller carry trade return. At the same time, counter-cyclical inflation reinforces the real channel by making the nominal exchange rate more volatile and pro-cyclical – this is exactly the same, purely nominal, channel described in Section 4.1.

### 4.4.2 Asymmetry in financial trade cost

Panel E of Table 3 presents a model in which it is more costly to invest into financial assets of country F than of country H: we set $\kappa^{HF} = 0.01$ and all other $\kappa$'s to zero. We study this asymmetry because Garleanu, Panageas, and Yu (2015) show that certain countries are more closed to foreign financial investment than others, and we show in our empirical analysis in Section 2 (Panel B of Table 2) that these same countries tend to have higher interest rates. In our model, F must have a higher interest rate to compensate investors from H who are paying a cost to invest in F. Additionally, the cost works in a similar way to symmetric costs in previous models – it brings the model closer to a no-trade Balassa-Samuelson world, which results in a positive correlation between the currency appreciation of a country, and that country's output and consumption growth.

As in the model with default, allowing for counter-cyclical inflation through a zero expected inflation monetary policy has a dual effect. On the one hand it improves risk sharing, which reduces the real exchange rate volatility and the carry trade profit. This happens because in any state of the world in which H received a relatively positive shock, while F received a relatively negative shock, F has a relatively higher inflation than H. Therefore, the real value of H-bonds (F-bonds) increases (decreases) which benefits (hurts) investors from F (H) holding H-bonds (F-bonds). This improves risk sharing. At the same time, as in the case of Section 4.1 and of asymmetry in default, inflation makes the nominal exchange rate more volatile which can reinforce the real channel for carry trade profit.

### 4.5 Two puzzles and quantitative limitations

There are two puzzles regarding exchange rates and risk sharing emphasized in studies of international finance. The first is the Backus and Smith (1993) puzzle regarding the correlation of exchange rates, the stochastic discount factor, and consumption. As shown in Backus and Smith (1993), complete markets imply that $\Delta er = m_H - m_F$. We follow Backus, Foresi, and Telmer (2001) and write an extended version of this relationship

$$\Delta er_{t+1} = m_{H,t+1} - m_{F,t+1} - \eta_{t+1} \tag{22}$$

where $\eta$ is a deviation from complete markets; $\eta = 0$ implies that markets are complete and each stochastic discount factor is unique. Recall that $er$ is the real strength of home currency.

As explained earlier, the Backus and Smith (1993) puzzle deals with the correlation of consumption growth and exchange rates. For example with CRRA utility, $m_{t+1} = -\gamma \Delta c_{t+1}$, which means that consumption growth in H is negatively related to currency appreciation in H – this is counterfactual. We have shown earlier that incomplete markets can help. To see this formally, take the covariances of $\Delta er_{t+1}$ and $m_{H,t+1} - m_{F,t+1}$:

$$\begin{aligned} \sigma_{\Delta er, m_H - m_F} &= \sigma_{m_H - m_F - \eta, m_H - m_F} = \sigma^2_{m_H - m_F} - \sigma_{m_H,\eta} + \sigma_{m_F,\eta} \\ &= \sigma^2_{m_H} + \sigma^2_{m_F} - 2\sigma_{m_H, m_F} - \sigma_{m_H,\eta} + \sigma_{m_F,\eta} \end{aligned} \tag{23}$$

If markets are complete and $\eta = 0$, then currency appreciation and the stochastic discount factor must be positively correlated. But if $\sigma_{m_H,\eta} > 0$ and $\sigma_{m_F,\eta} < 0$ then this is no longer the case – this is exactly what happens in our model.

Brandt, Cochrane, and Santa-Clara (2006) argue that either measured exchange rates are too smooth, or risk is shared very well across countries. To see their argument, we take the variance of both sides of equation (22):

$$\sigma^2_{\Delta er} = \sigma^2_{m_H} + \sigma^2_{m_F} - 2\sigma_{m_H, m_F} + \sigma^2_{\eta} - 2\sigma_{m_H,\eta} + 2\sigma_{m_F,\eta} \tag{24}$$

From domestic financial markets, we know that stochastic discount factors are very volatile (Hansen and Jagannathan (1991)) – the lower bound is around 0.4. However, exchange rates are not very volatile – roughly 1/4 as volatile as stochastic discount factors. If markets are complete and $\eta = 0$, then the only way for equation (24) to hold is for the two stochastic discount factors to be highly correlated. A high correlation of stochastic discount factors is equivalent to a large amount of risk sharing. However, CRRA utiltiy would imply the same degree of correlation in consumption growth – in the data consumption growth is only moderately correlated.

This argument need not hold if markets are incomplete, this point is also made by Bakshi, Cerrato, and Crosby (2015). For example, in our real model with default (first row of Panel D in Table 3), the standard deviations of the H and F stochastic discount factors are 0.227 and 0.18, the standard deviation of the exchange rate is 0.0365. If markets were complete ($\eta = 0$), these numbers would imply that the two stochastic discount factors should have a correlation above one. In the model, their correlation is only 0.109. This is because the term $\sigma^2_{\eta} - 2\sigma_{m_H,\eta} + 2\sigma_{m_F,\eta}$ in equation (24) is negative.

Figure 1 presents these results graphically. Each panel is a type of asymmetry: Volatility, Risk aversion, Size, or Other (Default and Cost). For the first three, we compare the complete market model with non-tradable shocks (circles) to the incomplete market model with nominal bonds and costs of trade (triangles). In complete market models, there is a perfect positive relationship between currency appreciation and the stochastic discount factor ratio; this leads to the Backus and Smith (1993) and Brandt, Cochrane, and Santa-Clara (2006) puzzles. Incomplete market models reverse this relationship, leading to a positive correlation – this helps with the Backus and Smith (1993). Furthermore, the spread in stochastic discount factor realizations is wider, while the spread in exchange rate realizations is narrower – this helps with the carry trade premium and the Brandt, Cochrane, and Santa-Clara (2006) puzzle.

One limitation of incomplete markets is that they may reduce the quantitative size of the carry trade. Lustig and Verdelhan (2015) use Euler equations to derive restrictions on $\eta$ and show that for a fairly large class of models, it is not possible for incompleteness alone to resolve the Backus and Smith (1993) and Brandt, Cochrane, and Santa-Clara (2006) puzzles, while simultaneously to match the return on the carry trade. Their restrictions on $\eta$ apply to some, but not all, of the models we consider; for example they do not apply when sovereign default is possible. In particular, our real model with default (first row of Panel D in Table 3) has a positive correlation between consumption growth and currency appreciation (Backus and Smith (1993) puzzle), exchange rate volatility roughly an order of magnitude below SDF volatility (Brandt, Cochrane, and Santa-Clara (2006) puzzle), and a positive carry trade return.[45]

# 5 Conclusion

We propose a fairly general model which can reproduce the failure of UIP and the profitability of the carry trade in multiple environments. When we consider some of the environments that have been proposed in the past (differences across countries in volatility, risk aversion, size, and loading on aggregate risk), we find that these are often at odds with the data along several dimensions. We show that allowing for market incompleteness can improve these models along one important dimension: the correlation between currency returns and (local) consumption or output growth. However, incomplete markets cannot help along other dimensions: the assumptions about the nature of the shocks (non-tradable in most models) and cross-country heterogeneity do not fit

[45] The carry trade return is still lower than the data (Sharpe ratio 0.1), however, we believe that this is because this calibration implies stochastic discount factors which are not volatile enough relative to the Hansen and Jagannathan (1991) bounds. For example, if the risk aversion rises to 30, then the stochastic discount factor becomes sufficiently volatile, and the carry trade Sharpe ratio rises to 0.4, without significantly changing the exchange rate volatility or correlation with consumption. While a risk aversion of 30 may be unrealistic, this exercise shows that having all three results in a single model is possible. Making the model dynamic would allow for more volatile stochastic discount factors without such high risk aversion, through the Bansal and Yaron (2004) long run risk channel.

the data. For this reason, we explore two alternative environments: cross-sectional differences in likelihood of default, and in barriers to financial trade. In our model, we show that these differences can drive carry trade profits, and that these differences are consistent with the data as well.

Our model is not perfect. First, its goal is a qualitative description of the carry trade and UIP, and while we believe it describes these phenomena well qualitatively, a quantitative description is the ultimate goal. Second, the model cannot match the higher consumption volatility in high interest rate countries. We believe that a dynamic, rather than a two-period, model will help along both dimensions.

# A Relation to other models

Consider a model where markets are complete and there is no trade due to perfect home bias ($\theta = 1$), thus output is equal to consumption. Utility is CRRA but risk aversion can be time varying. The logarithm of the real stochastic discount factor is $m_{t+1} = -\gamma_t \Delta c_{t+1}$. The logarithm of the nominal stochastic discount factor is $mn_{t+1} = m_{t+1} - i^H_{t+1} = -\gamma_t \Delta c_{t+1} - i^H_{t+1}$. Let $e_t$ be the log of foreign currency per unit of home, thus a high $e_t$ indicates a strong home currency.

From no arbitrage (for example see Backus and Smith (1993)), the currency appreciation is:

$$\Delta e_{t+1} = m^H_{t+1} - m^F_{t+1} - (i^H_{t+1} - i^F_{t+1}) = -\gamma^H_t \Delta c^H_{t+1} + \gamma^F_t \Delta c^F_{t+1} - (i^H_{t+1} - i^F_{t+1}). \tag{A1}$$

The nominal interest rate in either country is given by $r_{t+1} = -\log(\mathbb{E}_t[e^{m_{t+1} - i_{t+1}}])$. Assuming log normal shocks:

$$\begin{aligned} r_{t+1} &= -\mathbb{E}_t[m_{t+1}] + \mathbb{E}_t[i_{t+1}] - 0.5\sigma^2_t[m_{t+1}] - 0.5\sigma^2_t[i_{t+1}] + cov_t[m_{t+1}, i_{t+1}] && \text{(A2)} \\ &= \gamma_t \mathbb{E}_t[\Delta c_{t+1}] + \mathbb{E}_t[i_{t+1}] - 0.5\gamma^2_t \sigma^2_t[\Delta c_{t+1}] - 0.5\sigma^2_t[i_{t+1}] - \gamma_t cov_t[\Delta c_{t+1}, i_{t+1}]. && \text{(A3)} \end{aligned}$$

The interest rate differential between the two countries is:

$$\begin{aligned} r^F_{t+1} - r^H_{t+1} = \; & \gamma^F_t \mathbb{E}_t[\Delta c^F_{t+1}] - \gamma^H_t \mathbb{E}_t[\Delta c^H_{t+1}] \\ & -0.5\left((\gamma^F_t)^2 \sigma^2_t[\Delta c^F_{t+1}] - (\gamma^H_t)^2 \sigma^2_t[\Delta c^H_{t+1}]\right) \\ & +\mathbb{E}_t[i^F_{t+1}] - \mathbb{E}_t[i^H_{t+1}] \\ & -0.5\left(\sigma^2_t[i^F_{t+1}] - \sigma^2_t[i^H_{t+1}]\right) \\ & -\gamma^F_t cov_t[\Delta c^F_{t+1}, i^F_{t+1}] + \gamma^H_t cov_t[\Delta c^H_{t+1}, i^H_{t+1}]. \end{aligned} \tag{A4}$$

Define the carry trade return $r^{CT}_{t+1}$ as borrowing in H and investing in F, regardless of which country has a higher interest rate. The expected carry trade return is:

$$\begin{aligned} \mathbb{E}_t[r^{CT}_{t+1}] &= r^F_{t+1} - r^H_{t+1} - \mathbb{E}_t[\Delta e_{t+1}] \\ &= -0.5\left((\gamma^F_t)^2 \sigma^2_t[\Delta c^F_{t+1}] - (\gamma^H_t)^2 \sigma^2_t[\Delta c^H_{t+1}]\right) \\ &\quad -0.5\left(\sigma^2_t[i^F_{t+1}] - \sigma^2_t[i^H_{t+1}]\right) \\ &\quad -\gamma^F_t cov_t[\Delta c^F_{t+1}, i^F_{t+1}] + \gamma^H_t cov_t[\Delta c^H_{t+1}, i^H_{t+1}], \end{aligned} \tag{A5}$$

where the second equality follows from (A1) and (A4). Note that as in our model, expected differences in inflation play no role in carry trade profits because any differences in nominal interest rates are offset by expected currency movements. The unexpected carry trade profit depends only on the shocks to the exchange rate, since the interest rate differential is not a function of $t+1$ shocks:

$$\begin{aligned} r^{CT}_{t+1} - \mathbb{E}_t[r^{CT}_{t+1}] &= -(\Delta e_{t+1} - \mathbb{E}_t[\Delta e_{t+1}]) \\ &= \gamma^H_t \left(\Delta c^H_{t+1} - \mathbb{E}_t[\Delta c^H_{t+1}]\right) - \gamma^F_t \left(\Delta c^F_{t+1} - \mathbb{E}_t[\Delta c^F_{t+1}]\right) \\ &\quad + \left(i^H_{t+1} - \mathbb{E}_t[i^H_{t+1}\right) - \left(i^F_{t+1} - \mathbb{E}_t[i^F_{t+1}\right) \end{aligned} \tag{A6}$$

Now consider two of the leading explanations for carry trade profits: Verdelhan (2010) and Bansal and Shaliastovich (2012). Both models assume complete financial markets but perfect home bias (no trade). The former relies on time varying risk aversion, while the later on time varying consumption and inflation volatility. The former uses habit preferences to achieve differences in risk aversion; we believe differences in risk aversion are the main driving force. The latter uses differences in volatility together Epstein-Zin-Weil preferences, though we believe that differences in volatility, with a simpler utility function, would be sufficient for the qualitative results.

Let us start with Verdelhan (2010), where $i_t = 0$. The simplest way to gain intuition is to assume that $\sigma_t[\Delta c^F_{t+1}] = \sigma_t[\Delta c^H_{t+1}]$ and $\mathbb{E}_t[\Delta c^F_{t+1}] = \mathbb{E}_t[\Delta c^H_{t+1}] = g$. Country H has higher risk aversion than country F: $\gamma^H_t > \gamma^F_t$. In this case equations (A4), (A5), and (A6) reduce to:

$$\begin{aligned} r^F_{t+1} - r^H_{t+1} &= \left(\gamma^F_t - \gamma^H_t\right) \mathbb{E}_t[\Delta c_{t+1}] \\ &\quad 0.5 \left((\gamma^H_t)^2 - (\gamma^F_t)^2\right) \sigma^2_t[\Delta c_{t+1}] \end{aligned} \tag{A7}$$

$$\mathbb{E}_t[r^{CT}_{t+1}] = 0.5 \left((\gamma^H_t)^2 - (\gamma^F_t)^2\right) \sigma^2_t[\Delta c_{t+1}] \tag{A8}$$

$$r^{CT}_{t+1} - \mathbb{E}_t[r^{CT}_{t+1}] = \gamma^H_t \Delta c^H_{t+1} - \gamma^F_t \Delta c^F_{t+1} - (\gamma^H_t - \gamma^F_t)g \tag{A9}$$

If $\gamma^H_t > \gamma^F_t$ then this trade will always be profitable on average, from equation (A8). However, for it to be defined as the carry trade, the Foreign interest rate needs to be higher. This will happen if the second term in equation (A7) dominates; this is the precautionary saving channel and it will dominate if the risk aversion difference is sufficiently high. This model implies that the realized carry trade profits are highest when consumption growth in the carry target country (F) is low. Additionally, in a multi-period, dynamic model with habit, in order for H to have a higher risk aversion, H must have received recent negative shocks. This implies that the carry trade target country (F) is bigger.

In Bansal and Shaliastovich (2012), inflation is no longer zero and, like in our model, the carry trade profit can be attributed to one of two channels, one real and one nominal. Although Bansal and Shaliastovich (2012) employ Epstein-Zin-Weil preferences, the intuition is easiest to see with CRRA utility with a time-invariant coefficient of relative risk aversion. Additionally, for simplicity assume that $\mathbb{E}_t[\Delta c^F_{t+1}] = \mathbb{E}_t[\Delta c^H_{t+1}]$ but $\sigma_t[\Delta c^H_{t+1}] > \sigma_t[\Delta c^F_{t+1}]$. In this case equations (A4), (A5), and (A6) reduce to:

$$\begin{aligned} r^F_{t+1} - r^H_{t+1} &= 0.5\gamma^2\left(\sigma^2_t[\Delta c^H_{t+1}] - \sigma^2_t[\Delta c^F_{t+1}]\right) \\ &\quad +\mathbb{E}_t[i^F_{t+1}] - \mathbb{E}_t[i^H_{t+1}] \\ &\quad +0.5\left(\sigma^2_t[i^H_{t+1}] - \sigma^2_t[i^F_{t+1}]\right) \\ &\quad +\gamma\left(cov_t[\Delta c^H_{t+1}, i^H_{t+1}] - cov_t[\Delta c^F_{t+1}, i^F_{t+1}]\right) \end{aligned} \tag{A10}$$

$$\begin{aligned} \mathbb{E}_t[r^{CT}_{t+1}] &= 0.5\gamma^2\left(\sigma^2_t[\Delta c^H_{t+1}] - \sigma^2_t[\Delta c^F_{t+1}]\right) \\ &\quad +0.5\left(\sigma^2_t[i^H_{t+1}] - \sigma^2_t[i^F_{t+1}]\right) \\ &\quad +\gamma\left(cov_t[\Delta c^H_{t+1}, i^H_{t+1}] - cov_t[\Delta c^F_{t+1}, i^F_{t+1}]\right) \end{aligned} \tag{A11}$$

$$\begin{aligned} r^{CT}_{t+1} - \mathbb{E}_t[r^{CT}_{t+1}] &= \gamma\left(\Delta c^H_{t+1} - \Delta c^F_{t+1}\right) \\ &\quad +\left(i^H_{t+1} - i^F_{t+1}\right) - \left(\mathbb{E}_t[i^H_{t+1}] - \mathbb{E}_t[i^F_{t+1}]\right) \end{aligned} \tag{A12}$$

First consider the real side. If $\sigma_t[\Delta c^H_{t+1}] > \sigma_t[\Delta c^F_{t+1}]$ then country F has a higher interest rate and the carry trade of borrowing in H and investing in F is profitable. Thus, carry trade target countries should have a lower expected consumption growth volatility. Additionally, as in Verdelhan (2010), this model implies that the realized carry trade profits are highest when consumption growth in the carry target country (F) is low.

Next, consider the nominal side. All else equal, if $\sigma_t[i^H_{t+1}] > \sigma_t[i^F_{t+1}]$ then country F has a higher interest rate and the carry trade of borrowing in H and investing in F is profitable. Thus, carry trade target countries should have a lower expected inflation volatility. Additionally, as in our model, this model implies that the realized carry trade profits are highest when realized inflation in the target country (F) is low. In this model, carry trade profits can also come through lower (more negative) expected covariance of inflation with consumption growth in the target country. Finally, all else equal, the country with a higher expected inflation has a higher interest rate, however this has no effect on carry trade profits.

It is straightforward to map the intuition of Bansal and Shaliastovich (2012) and Verdelhan (2010) models into our two-period framework. To solve their models exactly, we would need to set $\theta = 1$ in the definition (3) of composite consumption good, so that there is no trade. However this is a stronger restriction than necessary. Instead, we can just switch from tradable to non-tradable shocks, but still allow for trade in tradable goods. For simplicity, we also assume that monetary policy achieves exactly zero inflation, thus only the real channel is at work. These results are presented in row one of Panels A (higher volatility in F) and B (higher risk aversion in F) of Table 3. In both cases, consistent with Bansal and Shaliastovich (2012) and Verdelhan (2010), country H has a higher interest rate, and the carry trade from F to H yields a positive expected profit. However, the strength of a country's currency is negatively correlated with its output shocks, implying that the carry trade is most profitable when the target country experiences negative shocks. Recall that in the data, as in our models with trade, a country's currency is *positively* correlated with its output shocks, and the carry trade is most profitable when the target country experiences *positive* shocks.

In Bansal and Shaliastovich (2012), the carry trade is profitable due to one or two channels. The real channel relies on asymmetry in volatility. This is exactly the same channel as in Tran (2013). The nominal channel relies on counter-cyclical inflation, which is related to the nominal channel in our model. However, unlike our model, Bansal and Shaliastovich (2012) assume an exogenous counter-cyclical process for inflation. In our models, inflation alone can lead to carry trade profits, but inflation is endogenously counter-cyclical.

# B Model details

This appendix provides details regarding of the incomplete market model described in Section3. Subsection B.1 formally derives the equilibrium and describes the numerical procedure we use. Subsection B.2 describes the equilibrium in an equivalent model where a complete set of Arrow Debreu Securities exists. Subsection B.3 proofs the inexistence of equilibrium in special version of our model in which only real bonds are available for trade.

## B.1 Derivation of equilibrium

The H household problem is

$$\max_{\{C^{\mathrm{T}}_{\mathrm{H},0}, C^{\mathrm{N}}_{\mathrm{H},0}, B^{\mathrm{H}}_{\mathrm{H}}, B^{\mathrm{F}}_{\mathrm{H}}, C^{\mathrm{T}}_{\mathrm{H},1,i}, C^{\mathrm{N}}_{\mathrm{H},1,i}\}_{i=1}^{K}} \left( C_{\mathrm{H},0}^{1-\rho} + \beta \mathbb{E}\left[ \widetilde{C}_{\mathrm{H},1}^{1-\gamma} \right]^{\frac{1-\rho}{1-\gamma}} \right)^{\frac{1}{1-\rho}}, \quad 0 < \beta < 1, \tag{B13}$$

where

$$\widetilde{C}_{\mathrm{H},1} = (C_{\mathrm{H},1,1}, \ldots, C_{\mathrm{H},1,K}) \tag{B14}$$

$$C_{\mathrm{H},0} = ((1-\theta)(C^{\mathrm{T}}_{\mathrm{H},0})^{\alpha} + \theta(C^{\mathrm{N}}_{\mathrm{H},0})^{\alpha})^{\frac{1}{\alpha}} \tag{B15}$$

$$C_{\mathrm{H},1,i} = \left((1-\theta)(C^{\mathrm{T}}_{\mathrm{H},1,i})^{\alpha} + \theta(C^{\mathrm{N}}_{\mathrm{H},1,i})^{\alpha}\right)^{\frac{1}{\alpha}}, \quad 0 < \theta < 1, \quad i = 1, \ldots, K, \tag{B16}$$

subject to the budget constraints

$$P^{\mathrm{T}}_{\mathrm{H},0} C^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0} C^{\mathrm{N}}_{\mathrm{H},0} + q_{\mathrm{H}} B^{\mathrm{H}}_{\mathrm{H}} + \frac{q_{\mathrm{F}}}{E_0} B^{\mathrm{F}}_{\mathrm{H}} = P^{\mathrm{T}}_{\mathrm{H},0} Y^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0} Y^{\mathrm{T}}_{\mathrm{H},0} \tag{B17}$$

$$P^{\mathrm{T}}_{\mathrm{H},1,i} C^{\mathrm{T}}_{\mathrm{H},1,i} + P^{\mathrm{N}}_{\mathrm{H},1,i} C^{\mathrm{N}}_{\mathrm{H},1,i} = P^{\mathrm{T}}_{\mathrm{H},1,i} Z^{T}_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1,i} Z^{N}_{\mathrm{H},i} Y^{\mathrm{N}}_{\mathrm{H},1} + B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{E_{1,i}} B^{\mathrm{F}}_{\mathrm{H}}, \tag{B18}$$

and the resource constraints

$$C^{\mathrm{N}}_{\mathrm{H},1} = Y^{\mathrm{N}}_{\mathrm{H},0} \tag{B19}$$

$$C^{\mathrm{N}}_{\mathrm{H},1,i} = Z^{N}_{\mathrm{H},i} Y^{\mathrm{N}}_{\mathrm{H},1}, \quad i = 1, \ldots, K. \tag{B20}$$

The expectation $\mathbb{E}^{\pi}[x] = \sum_{i=1}^{k} \pi_i x_i$, where $\pi_i$ denotes the probability of state $i$.

### Solution details

The Lagrangian is:

$$\mathcal{L} = (C_{\mathrm{H},0}^{1-\rho} + \beta \mathbb{E}^{\pi}[\widetilde{C}_{\mathrm{H},1}^{1-\gamma}]^{\frac{1-\rho}{1-\gamma}})^{\frac{1}{1-\rho}} + \lambda_0 (P^{\mathrm{T}}_{\mathrm{H},0} Y^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0} Y^{\mathrm{N}}_{\mathrm{H},0} - P^{\mathrm{T}}_{\mathrm{H},0} C^{\mathrm{T}}_{\mathrm{H},0} - P^{\mathrm{N}}_{\mathrm{H},0} C^{\mathrm{N}}_{\mathrm{H},0} - q_{\mathrm{H}} B^{\mathrm{H}}_{\mathrm{H}} - \frac{q_{\mathrm{F}}}{E_0} B^{\mathrm{F}}_{\mathrm{H}})$$

$$+ \beta \sum_{i=1}^{k} \pi_i \lambda_{1,i} (P^{\mathrm{T}}_{\mathrm{H},1,i} Z^{T}_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1,i} Z^{N}_{\mathrm{H},i} Y^{\mathrm{N}}_{\mathrm{H},1} + B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{E_{1,i}} B^{\mathrm{F}}_{\mathrm{H}} - P^{\mathrm{T}}_{\mathrm{H},1,i} C^{\mathrm{T}}_{\mathrm{H},1,i} - P^{\mathrm{N}}_{\mathrm{H},1,i} C^{\mathrm{N}}_{\mathrm{H},1,i})$$

The FOCs for consumption at time 0 yield

$$u^{\rho} C_{\mathrm{H},0}^{-\rho} (1-\theta)(C_{\mathrm{H},0}^{\mathrm{T}})^{\alpha-1} = \lambda_0 P_{\mathrm{H},0}^{\mathrm{T}} \tag{B21a}$$

$$u^{\rho} C_{\mathrm{H},0}^{-\rho} \theta (C_{\mathrm{H},0}^{\mathrm{N}})^{\alpha-1} = \lambda_0 P_{\mathrm{H},0}^{\mathrm{N}} \tag{B21b}$$

The FOCs for consumption at time 1 yield

$$u^{\rho} \mathbb{E}^{\pi} [C_{\mathrm{H},1}^{1-\gamma}]^{\frac{\gamma-\rho}{1-\gamma}} C_{\mathrm{H},1,i}^{-\gamma} \ (1-\theta)(C_{\mathrm{H},1,i}^{\mathrm{T}})^{\alpha-1} = \lambda_{1,i} P_{\mathrm{H},1,i}^{\mathrm{T}} \tag{B22a}$$

$$u^{\rho} \mathbb{E}^{\pi} [C_{\mathrm{H},1}^{1-\gamma}]^{\frac{\gamma-\rho}{1-\gamma}} (1-\gamma) C_{\mathrm{H},1,i}^{-\gamma} \ \theta (C_{\mathrm{H},1,i}^{\mathrm{N}})^{\alpha-1} = \lambda_{1,i} P_{\mathrm{H},1,i}^{\mathrm{N}} \tag{B22b}$$

The FOC for asset holdings yields

$$B_{\mathrm{H}}^{\mathrm{H}} : -\lambda_0 q_{\mathrm{H}} + \beta \sum_{i=1}^{k} \pi_i \lambda_{1,i} = 0 \Rightarrow q_{\mathrm{H}} = \mathbb{E}\left[\beta \frac{\lambda_1}{\lambda_0}\right]. \tag{B23}$$

Let us define nominal expenditures as $P_t C_t = P^T C^T + P^N C^N$. Using the FOC (B21) and (B22) we have

$$\begin{aligned} \lambda_t P_{\mathrm{H,t}} C_{\mathrm{H,t}} &= u^{\rho} \beta^t CEQ_t^{\gamma-\rho} C_{\mathrm{H,t}}^{1-\alpha-\gamma} \left((1\theta)(C_{\mathrm{H,t}}^{\mathrm{T}})^{\alpha} + \theta (C_{\mathrm{H,t}}^{\mathrm{N}})^{\alpha}\right) \\ &= u^{\rho} \beta^t CEQ_t^{\gamma-\rho} C_{\mathrm{H,t}}^{1-\gamma}, \end{aligned} \tag{B24}$$

where we defined $CEQ_0 \equiv C_{\mathrm{H},0}$ and $CEQ_1 \equiv \mathbb{E}^{\pi}[C_{\mathrm{H},1}^{1-\gamma}]^{\frac{1}{1-\gamma}}$. This yield the following expression for the lagrange multipliers $\lambda_t$, $t = 0, 1$:

$$\lambda_t = \beta^t \frac{u^{\rho} CEQ_t^{\gamma-\rho} C_{\mathrm{H,t}}^{-\gamma}}{P_{\mathrm{H,t}}}. \tag{B25}$$

From this expression we obtain that the nominal SDF $\widetilde{\mathbb{M}}_H^{\$}$ in country $H$ is given by

$$\widetilde{\mathbb{M}}_H^{\$} = \beta \left(\frac{\widetilde{C}_{\mathrm{H},1}}{C_{\mathrm{H},0}}\right)^{-\rho} \left(\frac{C_{\mathrm{H},1}}{\mathbb{E}[\widetilde{C}_{\mathrm{H},1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\rho-\gamma} \frac{P_{\mathrm{H},0}}{P_{\mathrm{H},1}}. \tag{B26}$$

The prices of the home and foreign bond in the home country are then

$$q_{\mathrm{H}} = \mathbb{E}\left[\widetilde{\mathbb{M}}^{\$}_{H}\right] \tag{B27}$$

$$q_{\mathrm{F}} = \mathbb{E}\left[\widetilde{\mathbb{M}}^{\$}_{H}\frac{E_0}{\widetilde{E}_1}\right] \tag{B28}$$

Substituting the lagrange multiplier (B25) in the FOC (B21) and (B22) we obtain the following demand functions

$$C^{\mathrm{T}}_{\mathrm{H,t}} = \left((1-\theta)\frac{P_{\mathrm{H,t}}}{P^{\mathrm{T}}_{\mathrm{H,t}}}\right)^{\frac{1}{1-\alpha}} C_{\mathrm{H,t}} \tag{B29}$$

$$C^{\mathrm{N}}_{\mathrm{H,t}} = \left(\theta\frac{P_{\mathrm{H,t}}}{P^{\mathrm{N}}_{\mathrm{H,t}}}\right)^{\frac{1}{1-\alpha}} C_{\mathrm{H,t}}. \tag{B30}$$

Substituting the demand equations in the definition of consumption basket (B15) and (B16) we obtain that the price index $P_t$ is given by

$$\begin{aligned} P_{\mathrm{J,0}} &= ((1-\theta)^{\frac{1}{1-\alpha}}(P^{\mathrm{T}}_{\mathrm{J,0}})^{\frac{\alpha}{\alpha-1}} + \theta^{\frac{1}{1-\alpha}}(P^{\mathrm{N}}_{\mathrm{J,0}})^{\frac{\alpha}{\alpha-1}})^{\frac{\alpha-1}{\alpha}}, \qquad J = H, F \\ P_{\mathrm{J},1,i} &= ((1-\theta)^{\frac{1}{1-\alpha}}(P^{\mathrm{T}}_{\mathrm{J},1,i})^{\frac{\alpha}{\alpha-1}} + \theta^{\frac{1}{1-\alpha}}(P^{\mathrm{N}}_{\mathrm{J},1,i})^{\frac{\alpha}{\alpha-1}})^{\frac{\alpha-1}{\alpha}}, \quad J = H, F, \quad i = 1, ..., K \end{aligned} \tag{B31}$$

The model solution can then be summarized into the following equations:

**Household budget constraints:**

$$P^{\mathrm{T}}_{\mathrm{H,0}}C^{\mathrm{T}}_{\mathrm{H,0}} + P^{\mathrm{N}}_{\mathrm{H,0}}C^{\mathrm{N}}_{\mathrm{H,0}} + q_{\mathrm{H}}B^{\mathrm{H}}_{\mathrm{H}} + \frac{q_{\mathrm{F}}}{E_0}B^{\mathrm{F}}_{\mathrm{H}} = P^{\mathrm{T}}_{\mathrm{H,0}}Y^{\mathrm{T}}_{\mathrm{H,0}} + P^{\mathrm{N}}_{\mathrm{H,0}}Y^{\mathrm{T}}_{\mathrm{H,0}} \tag{B32}$$

$$P^{\mathrm{T}}_{\mathrm{H},1,i}C^{\mathrm{T}}_{\mathrm{H},1,i} + P^{\mathrm{N}}_{\mathrm{H},1,i}C^{\mathrm{N}}_{\mathrm{H},1,i} = P^{\mathrm{T}}_{\mathrm{H},1,i}Z^{T}_{\mathrm{H},i}Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1,i}Z^{N}_{\mathrm{H},i}Y^{\mathrm{N}}_{\mathrm{H},1} + B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{E_{1,i}}B^{\mathrm{F}}_{\mathrm{H}}, \tag{B33}$$

$$P^{\mathrm{T}}_{\mathrm{F,0}}C^{\mathrm{T}}_{\mathrm{F,0}} + P^{\mathrm{N}}_{\mathrm{F,0}}C^{\mathrm{N}}_{\mathrm{F,0}} + q_{\mathrm{H}}E_0B^{\mathrm{H}}_{\mathrm{F}} + q_{\mathrm{F}}B^{\mathrm{F}}_{\mathrm{F}} = P^{\mathrm{T}}_{\mathrm{F,0}}Y^{\mathrm{T}}_{\mathrm{F,0}} + P^{\mathrm{N}}_{\mathrm{F,0}}Y^{\mathrm{T}}_{\mathrm{F,0}} \tag{B34}$$

$$P^{\mathrm{T}}_{\mathrm{F},1,i}C^{\mathrm{T}}_{\mathrm{F},1,i} + P^{\mathrm{N}}_{\mathrm{F},1,i}C^{\mathrm{N}}_{\mathrm{F},1,i} = P^{\mathrm{T}}_{\mathrm{F},1,i}Z^{T}_{\mathrm{F},i}Y^{\mathrm{T}}_{\mathrm{F},1} + P^{\mathrm{N}}_{\mathrm{F},1,i}Z^{N}_{\mathrm{F},i}Y^{\mathrm{N}}_{\mathrm{F},1} + B^{\mathrm{H}}_{\mathrm{F}}E_{1,i} + B^{\mathrm{F}}_{\mathrm{F}} \tag{B35}$$

**Market clearing for non-tradable goods:**

$$C^{\mathrm{N}}_{\mathrm{H},0} = Y^{\mathrm{N}}_{\mathrm{H},0} \tag{B36}$$

$$C^{\mathrm{N}}_{\mathrm{H},1,i} = Z^{N}_{\mathrm{H},i} Y^{\mathrm{N}}_{\mathrm{H},1,i} \tag{B37}$$

$$C^{\mathrm{N}}_{\mathrm{F},0} = Y^{\mathrm{N}}_{\mathrm{F},0} \tag{B38}$$

$$C^{\mathrm{N}}_{\mathrm{F},1,i} = Z^{N}_{\mathrm{F},i} Y^{\mathrm{N}}_{\mathrm{F},1,i} \tag{B39}$$

**Market Clearing for tradable goods:**

$$C^{\mathrm{T}}_{\mathrm{H},0} + C^{\mathrm{T}}_{\mathrm{F},0} = Y^{\mathrm{T}}_{\mathrm{H},0} + Y^{\mathrm{T}}_{\mathrm{F},0} \tag{B40}$$

$$C^{\mathrm{T}}_{\mathrm{H},1,i} + C^{\mathrm{T}}_{\mathrm{F},1,i} = Z^{T}_{\mathrm{H},i} Y^{\mathrm{T}}_{\mathrm{H},1,i} + Z^{T}_{\mathrm{F},i} Y^{\mathrm{T}}_{\mathrm{F},1,i} \tag{B41}$$

**Market clearing for financial assets**

$$B^{\mathrm{H}}_{\mathrm{H}} = -B^{\mathrm{H}}_{\mathrm{F}} \tag{B42}$$

$$B^{\mathrm{F}}_{\mathrm{H}} = -B^{\mathrm{F}}_{\mathrm{F}} \tag{B43}$$

**Inter-termporal optimality conditions (Euler Equations)**

$$q_H = \mathbb{E}\left[\widetilde{\mathbb{M}}^{\$}_{H}\right] \tag{B44}$$

$$q_F = \mathbb{E}\left[\widetilde{\mathbb{M}}^{\$}_{H} \frac{E_0}{\widetilde{E}_1}\right] \tag{B45}$$

$$q_H = \mathbb{E}\left[\widetilde{\mathbb{M}}^{\$}_{F} \frac{\widetilde{E}_1}{E_0}\right], \tag{B46}$$

$$q_F = \mathbb{E}\left[\widetilde{\mathbb{M}}^{\$}_{F}\right] \tag{B47}$$

**Intra-temporal optimality onditions**

$$\frac{P^{\mathrm{N}}_{\mathrm{H},0}}{P^{\mathrm{T}}_{\mathrm{H},0}} = \frac{\theta}{1-\theta}\left(\frac{C^{\mathrm{T}}_{\mathrm{H},0}}{Y^{\mathrm{N}}_{\mathrm{H},0}}\right)^{1-\alpha}$$

$$\frac{P^{\mathrm{N}}_{\mathrm{H},1,i}}{P^{\mathrm{T}}_{\mathrm{H},1,i}} = \frac{\theta}{1-\theta}\left(\frac{C^{\mathrm{T}}_{\mathrm{H},1,i}}{Z^{N}_{\mathrm{H},i} Y^{\mathrm{N}}_{\mathrm{H},1}}\right)^{1-\alpha}$$

**Money supply**

$$
\begin{aligned}
M_{\mathrm{H},0} &= P^{\mathrm{T}}_{\mathrm{H},0}Y^{\mathrm{T}}_{\mathrm{H},0} + P^{\mathrm{N}}_{\mathrm{H},0}Y^{\mathrm{N}}_{\mathrm{H},0} && \text{(B48)}\\
M_{\mathrm{F},0} &= P^{\mathrm{T}}_{\mathrm{F},0}Y^{\mathrm{T}}_{\mathrm{F},0} + P^{\mathrm{N}}_{\mathrm{F},0}Y^{\mathrm{N}}_{\mathrm{F},0} && \text{(B49)}\\
M_{\mathrm{H},1} &= P^{\mathrm{T}}_{\mathrm{H},1}Z^{T}_{\mathrm{H}}Y^{\mathrm{T}}_{\mathrm{H},1} + P^{\mathrm{N}}_{\mathrm{H},1}Z^{N}_{\mathrm{H}}Y^{\mathrm{N}}_{\mathrm{H},1} && \text{(B50)}\\
M_{\mathrm{F},1} &= P^{\mathrm{T}}_{\mathrm{F},1}Z^{T}_{\mathrm{F}}Y^{\mathrm{T}}_{\mathrm{F},1} + P^{\mathrm{N}}_{\mathrm{F},1}Z^{N}_{\mathrm{F}}Y^{\mathrm{N}}_{\mathrm{F},1} && \text{(B51)}
\end{aligned}
$$

**No arbitrage exchange rate**

$$
\begin{aligned}
E_0 &= \frac{P^{\mathrm{T}}_{\mathrm{F},0}}{P^{\mathrm{T}}_{\mathrm{H},0}} && \text{(B52)}\\
E_{1,i} &= \frac{P^{\mathrm{T}}_{\mathrm{F},1,i}}{P^{\mathrm{T}}_{\mathrm{H},1,i}} && \text{(B53)}
\end{aligned}
$$

Without loss of generality, we normalize to 1 the time 0 prices of tradable: $P^{\mathrm{T}}_{\mathrm{H},0} = P^{\mathrm{T}}_{\mathrm{H},0} = 1$, leaving 24 unknowns that are determined by the above 24 equations. Using (i) the intra-temporal conditions at time 1; (ii) the budget constrain at time 1; (ii) the market-clearing condition for non-tradable goods, and (iv) the money supply equation at time 1, we can solve for $P^{\mathrm{T}}_{\mathrm{H},1}$ as follows

$$
P^{\mathrm{T}}_{\mathrm{H},1} = \frac{(1-\theta)M_{\mathrm{H},1} - \theta\left(B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{E_1}B^{\mathrm{F}}_{\mathrm{H}}\right)\left(\frac{Z^{N}_{\mathrm{H}}Y^{\mathrm{N}}_{\mathrm{H},1}}{C^{\mathrm{T}}_{\mathrm{H},1}}\right)^{\alpha}}{Z^{T}_{\mathrm{H}}Y^{\mathrm{T}}_{\mathrm{H},1}\left(1-\theta+\theta\left(\frac{Z^{N}_{\mathrm{H}}Y^{\mathrm{N}}_{\mathrm{H},1}}{C^{\mathrm{T}}_{\mathrm{H},1}}\right)^{\alpha}\right)}. \tag{B54}
$$

Following the same steps for country F we have

$$
P^{\mathrm{T}}_{\mathrm{F},1} = \frac{(1-\theta)M_{\mathrm{F},1} - \theta\left(B^{\mathrm{H}}_{\mathrm{F}}E_1 + B^{\mathrm{F}}_{\mathrm{F}}\right)\left(\frac{Z^{N}_{\mathrm{F}}Y^{\mathrm{N}}_{\mathrm{F},1}}{C^{\mathrm{T}}_{\mathrm{F},1}}\right)^{\alpha}}{Z^{T}_{\mathrm{F}}Y^{\mathrm{T}}_{\mathrm{F},1}\left(1-\theta+\theta\left(\frac{Z^{N}_{\mathrm{F}}Y^{\mathrm{N}}_{\mathrm{F},1}}{C^{\mathrm{T}}_{\mathrm{F},1}}\right)^{\alpha}\right)}. \tag{B55}
$$

Equations (B54) and (B55), as well as the budget constraints (B33) and (B35) gives a system of 4 equation and 4 unknowns, which, *given* $B^{\mathrm{H}}_{\mathrm{H}}, B^{\mathrm{F}}_{\mathrm{H}}, q_H, q_F$, can be solved for $C^{\mathrm{T}}_{\mathrm{H},1}$ $C^{\mathrm{T}}_{\mathrm{F},1}$ $P^{\mathrm{T}}_{\mathrm{H},1}$ $P^{\mathrm{T}}_{\mathrm{F},1}$.

Following the same steps as above for time zero, we obtain a similar system of 4 equations and 4 unknowns: $C^{\mathrm{T}}_{\mathrm{H},0}$ $C^{\mathrm{T}}_{\mathrm{F},0}$ $P^{\mathrm{T}}_{\mathrm{H},0}$ $P^{\mathrm{T}}_{\mathrm{F},0}$, *given* $B^{\mathrm{H}}_{\mathrm{H}}, B^{\mathrm{F}}_{\mathrm{H}}, q_H, q_F$. Therefore, all equilibrium quantities and prices can be expressed in terms of the 4 unknowns: $B^{\mathrm{H}}_{\mathrm{H}}, B^{\mathrm{F}}_{\mathrm{H}}, q_H, q_F$. These four quantities are determined by the Euler equations (B44)–(B47). To solve for an equilibrium we conjecture a four-tuple $B^{\mathrm{H}}_{\mathrm{H}}, B^{\mathrm{F}}_{\mathrm{H}}, q_H, q_F$, determine the remaining variables, construct the corresponding SDF, and then verify whether the conjecture satisfies the Euler equation. We update the conjecture based on the Euler equation error and iterate until convergence.

## B.2 Complete markets

In this appendix we describes the equilibrium in a complete market version of the model described in Section 3.

Let us suppose that there exists a complete set of $K$ Arrow-Debreu (AD) securities that pay one unit of tradable good in state $i = 1, \ldots, K$ and zero otherwise. The First Welfare Theorem implies that we can solve the social planner's problem. Suppose the planner puts Pareto weights of $\lambda$ and $1-\lambda$ on H and F. The planner's problem is:

$$\begin{aligned}
&\max_{C^{\mathrm{T}}_{\mathrm{H},0}, C^{\mathrm{N}}_{\mathrm{H},0}, C^{\mathrm{T}}_{\mathrm{H},1}, C^{\mathrm{N}}_{\mathrm{H},1}} \lambda U_H + (1-\lambda) U_F \qquad s.t.\\
&C^{\mathrm{T}}_{\mathrm{H},0} + C^{\mathrm{T}}_{\mathrm{F},0} = Y^{\mathrm{T}}_{\mathrm{H},0} + Y^{\mathrm{T}}_{\mathrm{F},0}\\
&C^{\mathrm{T}}_{\mathrm{H},1} + C^{\mathrm{T}}_{\mathrm{F},1} = Z^{T}_{\mathrm{H}} Y^{\mathrm{T}}_{\mathrm{H},1} + Z^{T}_{\mathrm{F}} Y^{\mathrm{T}}_{\mathrm{F},1}\\
&U_H = \left( C^{1-\rho_H}_{\mathrm{H},0} + \beta_H \mathbb{E}\left[ C^{1-\gamma_H}_{\mathrm{H},1} \right]^{\frac{1-\rho_H}{1-\gamma_H}} \right)^{\frac{1}{1-\rho_H}}\\
&U_F = \left( C^{1-\rho_F}_{\mathrm{F},0} + \beta_F \mathbb{E}\left[ C^{1-\gamma_F}_{\mathrm{F},1} \right]^{\frac{1-\rho_F}{1-\gamma_F}} \right)^{\frac{1}{1-\rho_F}}\\
&C_{\mathrm{H,t}} = \left( (1-\theta)(C^{\mathrm{T}}_{\mathrm{H,t}})^{\alpha} + \theta (C^{\mathrm{N}}_{\mathrm{H,t}})^{\alpha} \right)^{\frac{1}{\alpha}}\\
&C_{\mathrm{F,t}} = \left( (1-\theta)(C^{\mathrm{T}}_{\mathrm{F,t}})^{\alpha} + \theta (C^{\mathrm{N}}_{\mathrm{F,t}})^{\alpha} \right)^{\frac{1}{\alpha}}
\end{aligned} \tag{B56}$$

The FOC with respect to $C^{\mathrm{T}}_{\mathrm{H},0}$ and $C^{\mathrm{T}}_{\mathrm{F},0}$ is:

$$\lambda U_H^{\rho_H} (C_{\mathrm{H},0})^{1-\rho_H-\alpha} (C^{\mathrm{T}}_{\mathrm{H},0})^{\alpha-1} = (1-\lambda) U_F^{\rho_F} (C_{\mathrm{F},0})^{1-\rho_F-\alpha} (C^{\mathrm{T}}_{\mathrm{F},0})^{\alpha-1} \tag{B57}$$

The FOC with respect to $C^{\mathrm{T}}_{\mathrm{H},1}$ and $C^{\mathrm{T}}_{\mathrm{F},1}$ is:

$$\begin{aligned}
& \lambda U_H^{\rho_H} \beta \mathbb{E}[(C_{\mathrm{H},1})^{1-\gamma_H}]^{\frac{\gamma_H-\rho_H}{1-\gamma_H}} (C_{\mathrm{H},1})^{1-\gamma_H-\alpha} (C^{\mathrm{T}}_{\mathrm{H},1})^{\alpha-1}\\
= \;& (1-\lambda) U_F^{\rho_F} \beta \mathbb{E}[(C_{\mathrm{F},1})^{1-\gamma_F}]^{\frac{\gamma_F-\rho_F}{1-\gamma_F}} (C_{\mathrm{F},1})^{1-\gamma_F-\alpha} (C^{\mathrm{T}}_{\mathrm{F},1})^{\alpha-1}
\end{aligned} \tag{B58}$$

The above are $K+1$ equations with $K+1$ unknowns.

For a given $\lambda$, we solve these $K+1$ equations numerically for quantities. The next step is to find the $\lambda$ that is consistent with a decentralized equilibrium. To do this we solve for prices and find the $\lambda$ which ensures that the present value of each country's consumption is equal to the present value of its production.

From the previous section we have:

$$\frac{P^{\mathrm{N}}_{\mathrm{H,t}}}{P^{\mathrm{T}}_{\mathrm{H,t}}} = \frac{\theta}{1-\theta}\left(\frac{C^{\mathrm{T}}_{\mathrm{H,t}}}{C^{\mathrm{N}}_{\mathrm{H,t}}}\right)^{1-\alpha} \tag{B59}$$

and

$$P_{\mathrm{H,t}} = ((1-\theta)^{\frac{1}{1-\alpha}}(P^{\mathrm{T}}_{\mathrm{H,t}})^{\frac{\alpha}{\alpha-1}} + \theta^{\frac{1}{1-\alpha}}(P^{\mathrm{N}}_{\mathrm{H,t}})^{\frac{\alpha}{\alpha-1}})^{\frac{\alpha-1}{\alpha}}, \tag{B60}$$

and analogous equations for F. Without loss of generality, we can normalize $P_{\mathrm{H,t}} = P_{\mathrm{F,t}} = 1$. Since markets are complete, quantities will be unaffected by monetary policy, and there is always a monetary policy that can achieve this price level. From the above equations, for any $\lambda$ we can solve for quantities and then use them to solve for the tradable and non-tradable prices.

Finally, we need to solve for the Arrow-Debreu prices. To do so, consider the decentralized problem of a household choosing how many Arrow-Debreu securities $s_i$ of each type $i$ to buy at a price $q_i$:

$$\begin{aligned}
&\max_{s_i, C^{\mathrm{T}}_{\mathrm{H},0}, C^{\mathrm{T}}_{\mathrm{H},1}} \left(C_{\mathrm{H},0}^{1-\rho_H} + \beta_H \mathbb{E}\left[C_{\mathrm{H},1}^{1-\gamma_H}\right]^{\frac{1-\rho_H}{1-\gamma_H}}\right)^{\frac{1}{1-\rho_H}} \qquad s.t.\\
&C_{\mathrm{H,t}} = \left((1-\theta)(C^{\mathrm{T}}_{\mathrm{H,t}})^{\alpha} + \theta(C^{\mathrm{N}}_{\mathrm{H,t}})^{\alpha}\right)^{\frac{1}{\alpha}}\\
&C^{\mathrm{T}}_{\mathrm{H},0}P^{\mathrm{T}}_{\mathrm{H},0} + C^{\mathrm{N}}_{\mathrm{H},0}P^{\mathrm{N}}_{\mathrm{H},0} + \textstyle\sum_{i=1,K} s_i q_i = Y_0\\
&C^{\mathrm{T}}_{\mathrm{H},1,i}P^{\mathrm{T}}_{\mathrm{H},1,i} + C^{\mathrm{N}}_{\mathrm{H},1,i}P^{\mathrm{N}}_{\mathrm{H},1,i} - s_i = Y_1
\end{aligned} \tag{B61}$$

The FOC with respect to $s_i$ gives the nominal Arrow-Debreu price:

$$q_H = \pi\beta\left(\frac{C_{\mathrm{H},1}}{\mathbb{E}[(C_{\mathrm{H},1})^{1-\gamma_H}]^{\frac{1}{1-\gamma_H}}}\right)^{\rho_H-\gamma_H}\left(\frac{C_{\mathrm{H},1}}{C_{\mathrm{H},0}}\right)^{1-\rho_H-\alpha}\left(\frac{C^{\mathrm{T}}_{\mathrm{H},1}}{C^{\mathrm{T}}_{\mathrm{H},0}}\right)^{\alpha-1}\frac{P^{\mathrm{T}}_{\mathrm{H},0}}{P^{\mathrm{T}}_{\mathrm{H},1}} \tag{B62}$$

where $\pi$ is the probability.

Given these prices we compute the present value of all consumption and output in H:

$$\begin{aligned} PVC &= C^{\mathrm{T}}_{\mathrm{H},0}P^{\mathrm{T}}_{\mathrm{H},0} + C^{\mathrm{N}}_{\mathrm{H},0}P^{\mathrm{N}}_{\mathrm{H},0} + \textstyle\sum_{i=1,K} q_i \left( C^{\mathrm{T}}_{\mathrm{H},1,i}P^{\mathrm{T}}_{\mathrm{H},1,i} + C^{\mathrm{N}}_{\mathrm{H},1,i}P^{\mathrm{N}}_{\mathrm{H},1,i}\right) \\ PVY &= Y^{\mathrm{T}}_{\mathrm{H},0}P^{\mathrm{T}}_{\mathrm{H},0} + Y^{\mathrm{N}}_{\mathrm{H},0}P^{\mathrm{N}}_{\mathrm{H},0} + \textstyle\sum_{i=1,K} q_i \left( Z^{T}_{\mathrm{H}}Y^{\mathrm{T}}_{\mathrm{H},1,i}P^{\mathrm{T}}_{\mathrm{H},1,i} + Z^{N}_{\mathrm{H}}Y^{\mathrm{N}}_{\mathrm{H},1,i}P^{\mathrm{N}}_{\mathrm{H},1,i}\right) \end{aligned} \tag{B63}$$

If $PVC < PVY$ we adjust $\lambda$ up, if $PVC > PVY$ we adjust $\lambda$ down. We then resolve for quantities and prices. This process continues until $PVC = PVY$.

By combining the inter-temporal pricing equations with the equation for $q$, we can rewrite the nominal Arrow-Debreu price as a function of the stochastic discount factor and inflation:

$$q_H = \pi\beta \left( \frac{C_{\mathrm{H},1}}{\mathbb{E}[(C_{\mathrm{H},1})^{1-\gamma_H}]^{\frac{1}{1-\gamma_H}}} \right)^{\rho_H - \gamma_H} \left( \frac{C_{\mathrm{H},1}}{C_{\mathrm{H},0}} \right)^{-\rho_H} \frac{P_{\mathrm{H},0}}{P_{\mathrm{H},1}} = M_H \frac{P_{\mathrm{H},0}}{P_{\mathrm{H},1}} \tag{B64}$$

Finally, the real and nominal Home currency appreciations are:

$$\begin{aligned} \tfrac{E_1}{E_0} &= \tfrac{q_H}{q_F} = \tfrac{M_H}{M_F} \left( \tfrac{P_{\mathrm{H},0}}{P_{\mathrm{H},1}} / \tfrac{P_{\mathrm{F},0}}{P_{\mathrm{F},1}} \right) \\ \tfrac{ER_1}{ER_0} &= \tfrac{q_H}{q_F} \left( \tfrac{P_{\mathrm{F},0}}{P_{\mathrm{F},1}} / \tfrac{P_{\mathrm{H},0}}{P_{\mathrm{H},1}} \right) = \tfrac{M_H}{M_F} \end{aligned} \tag{B65}$$

### B.3 No solution for the case of zero realized inflation

In this appendix we prove the inexistence of an equilibrium when monetary policy targets zero realized inflation, in a special case of the model developed in Section 3.

Let us consider the case of a log utility households where the H and F countries are perfectly symmetric ex-ante and the composite consumption good is represented by a Cobb-Douglas aggregator, i.e., $\alpha \to 0$ in (3) For $\alpha \to 0$, the price index (15) becomes

$$P_{\mathrm{H,t}} = \kappa \left(P^{\mathrm{T}}_{\mathrm{H,t}}\right)^{1-\theta} \left(P^{\mathrm{N}}_{\mathrm{H,t}}\right)^{\theta}, \quad \kappa \equiv \left(\frac{1}{1-\theta}\right)^{1-\theta} \left(\frac{1}{\theta}\right)^{\theta}. \tag{B66}$$

Without loss of generality we assume that $P_{\mathrm{H},0} = P_{\mathrm{F},0}$. Zero realized inflation implies that $P_{\mathrm{H},1} = P_{\mathrm{F},1} = 1$. From the first order conditions with respect to consumption, and using the resource constraint $C^{\mathrm{N}}_{\mathrm{H},1} = Y^{\mathrm{N}}_{\mathrm{H},1}$, we have

$$\frac{P^{\mathrm{N}}_{\mathrm{H},1}}{P^{\mathrm{T}}_{\mathrm{H},1}} = \frac{\theta}{1-\theta} \frac{C^{\mathrm{T}}_{\mathrm{H},1}}{Y^{\mathrm{N}}_{\mathrm{H},1}} \tag{B67}$$

Using the time-1 budget constraints of the H and F household, impose the assumption of zero realized inflation $P_{\mathrm{H},1} = P_{\mathrm{F},1} = 1$ and the optimality condition (B67) we can express the time-1 consumption of T good in H and F good as follows

$$C^{\mathrm{T}}_{\mathrm{H},1} = Z_{\mathrm{H}} Y^{\mathrm{T}}_{\mathrm{H},1} + \frac{1}{P^{\mathrm{T}}_{\mathrm{H},1}} B^{\mathrm{H}}_{\mathrm{H}} + \frac{1}{P^{\mathrm{T}}_{\mathrm{F},1}} B^{\mathrm{F}}_{\mathrm{H}} = \left(P^{\mathrm{T}}_{\mathrm{H},1}\right)^{1/\theta} Y^{\mathrm{N}}_{\mathrm{H},1} (1-\theta)^{1/\theta} \tag{B68}$$

$$C^{\mathrm{T}}_{\mathrm{F},1} = Z_{\mathrm{F}} Y^{\mathrm{T}}_{\mathrm{F},1} + \frac{1}{P^{\mathrm{T}}_{\mathrm{H},1}} B^{\mathrm{H}}_{\mathrm{F}} + \frac{1}{P^{\mathrm{T}}_{\mathrm{F},1}} B^{\mathrm{F}}_{\mathrm{F}} = \left(P^{\mathrm{T}}_{\mathrm{F},1}\right)^{1/\theta} Y^{\mathrm{N}}_{\mathrm{F},1} (1-\theta)^{1/\theta} \tag{B69}$$

where we used the fact that the nominal exchange rate $E_1 = P^{\mathrm{T}}_{\mathrm{F},1}/P^{\mathrm{T}}_{\mathrm{H},1}$. By symmetry, $B^{\mathrm{H}}_{\mathrm{H}} = -B^{\mathrm{F}}_{\mathrm{H}}$ and by market clearing $B^{\mathrm{H}}_{\mathrm{H}} = -B^{\mathrm{H}}_{\mathrm{F}}$. Hence we define $B \equiv B^{\mathrm{H}}_{\mathrm{H}}$ from which it follows that $B^{\mathrm{F}}_{\mathrm{H}} = -B$, $B^{\mathrm{H}}_{\mathrm{F}} = -B$, and $B^{\mathrm{F}}_{\mathrm{F}} = B$. Imposing $q_{\mathrm{H}} = q_{\mathrm{F}}$ and $E_1 = 1$ in the Euler equations we obtain

$$\mathbb{E}\left[\frac{(P^{\mathrm{T}}_{\mathrm{H},1})^{\frac{1-\theta}{\theta}}}{Y^{\mathrm{N}}_{\mathrm{H},1}}\right] = \mathbb{E}\left[\frac{(P^{\mathrm{T}}_{\mathrm{H},1})^{\frac{1}{\theta}}(P^{\mathrm{T}}_{\mathrm{F},1})^{-1}}{Y^{\mathrm{N}}_{\mathrm{H},1}}\right] \tag{B70}$$

$$\mathbb{E}\left[\frac{(P^{\mathrm{T}}_{\mathrm{F},1})^{\frac{1-\theta}{\theta}}}{Y^{\mathrm{N}}_{\mathrm{F},1}}\right] = \mathbb{E}\left[\frac{(P^{\mathrm{T}}_{\mathrm{F},1})^{\frac{1}{\theta}}(P^{\mathrm{T}}_{\mathrm{H},1})^{-1}}{Y^{\mathrm{N}}_{\mathrm{F},1}}\right] \tag{B71}$$

By using symmetry, imposing, wlog, that $Y^{\mathrm{N}}_{\mathrm{H},1} = Y^{\mathrm{N}}_{\mathrm{F},1} = 1$, and adding up (B68) and (B69), we can characterize the equilibrium as the solution of the following system of two non-linear equation in the two unknown $B$ and $P^{\mathrm{T}}_{\mathrm{H},1}$

$$\left(P^{\mathrm{T}}_{\mathrm{H},1}\right)^{-1/\theta} (1-\theta)^{1/\theta} = Z_{\mathrm{H}} + B\left(\frac{1}{P^{\mathrm{T}}_{\mathrm{H},1}} - \frac{1}{P^{\mathrm{T}}_{\mathrm{F},1}}\right) \tag{B72}$$

$$\mathbb{E}\left[(P^{\mathrm{T}}_{\mathrm{H},1})^{\frac{1-\theta}{\theta}}\right] = \mathbb{E}\left[(P^{\mathrm{T}}_{\mathrm{H},1})^{\frac{1}{\theta}}(P^{\mathrm{T}}_{\mathrm{F},1})^{-1}\right] \tag{B73}$$

where

$$P^{\mathrm{T}}_{\mathrm{F},1} = \left((1-\theta)^{-1/\theta}(Z_{\mathrm{H}} + Z_{\mathrm{F}}) - (P^{\mathrm{T}}_{\mathrm{H},1})^{-1/\theta}\right)^{-\theta}. \tag{B74}$$

We claim that equation (B73) can be satisfied only in states for which $Z_{\mathrm{H}} = Z_{\mathrm{F}}$. To see this, note that for states in which $Z_{\mathrm{H}} = Z_{\mathrm{F}}$, by symmetry, $P^{\mathrm{T}}_{\mathrm{H},1} = P^{\mathrm{T}}_{\mathrm{F},1}$. Suppose instead that there are two states on which $Z_{\mathrm{H}}$ and $Z_{\mathrm{F}}$ take different values, e.g., $a = (Z_{\mathrm{H}} = 1+\epsilon, Z_{\mathrm{F}} = 1-\epsilon)$ or $b = (Z_{\mathrm{H}} = 1-\epsilon, Z_{\mathrm{F}} = 1+\epsilon)$, with equal probability. Let $x \equiv P^{\mathrm{T}}_{\mathrm{H},1}$ and $y \equiv P^{\mathrm{T}}_{\mathrm{F},1}$ in state $a$, where $y$

is obtained from (B74). The system (B72)-(B73) can then be written as

$$x^{-1/\theta}(1-\theta)^{1/\theta} = 1+\epsilon+B\left(\frac{1}{x}-\left(2(1-\theta)^{-1/\theta}-x^{-1/\theta}\right)^{\theta}\right) \tag{B75}$$

$$x^{\frac{1-\theta}{\theta}}+y^{\frac{1-\theta}{\theta}} = x^{1/\theta}y^{-1}+y^{1/\theta}x^{-1}, \tag{B76}$$

where $y=\left(2(1-\theta)^{-1/\theta}-x^{-1/\theta}\right)^{-\theta}$. Let $z\equiv\frac{x}{1-\theta}$, then the above system can be written as

$$z^{-1/\theta} = 1+\epsilon+B\left(\frac{1}{z}-\left(2-z^{-1/\theta}\right)^{\theta}\right)\frac{1}{1-\theta} \tag{B77}$$

$$z^{\frac{1-\theta}{\theta}}+\left(2-z^{-1/\theta}\right)^{\theta-1} = z^{1/\theta}+(2-z)^{-1}+z^{-1}(2-z)^{1/\theta}. \tag{B78}$$

The only solution to equation (B78) is $z=1$. To see this, let $q\equiv z^{-1/\theta}(2-z^{-1/\theta})^{-1}$ in (B78), which can then be rewritten as

$$1+q^{1-\theta}=q^{-\theta}+q. \tag{B79}$$

Let us define the function $f(q)=1-q+q^{1-\theta}-q^{-\theta}$. Direct inspection shows that $f(1)=0$ and that a global maximum is achieved at $q=1$. Therefore, $q=1$ is the only solution to (B78). From $q\equiv z^{-1/\theta}(2-z^{-1/\theta})^{-1}$, $q=1$ implies $z=1$. But $z=1$ can be a solution for (B77) only if $\epsilon=0$. Therefore we showed that an equilibrium with zero realized inflation is impossible unless both countries are perfectly identical ex-post, i.e., $\epsilon=0$.

# C Data

Our currency data set consists of daily observations of US-dollar based spot and forward exchange rates for 1 month, 3 months and 1 year maturities, for 40 countries. These countries are as follows: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, France, Germany, Hong Kong, Hungary, Iceland, Indonesia, India, Ireland, Italy, Japan, Korea, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Saudi Arabia,Singapore, Slovak Republic, South Africa, Spain, Sweden, Switzerland, Taiwan, Thailand, Turkey, UAE, UK. We extract the forward rates and spot rates from DataStream. Our data gathering procedure follows the one used by Hassan and Mano (2015). We gather spot rates and forward rates from four sources of data DataStream: World Markets PLC/Reuters (WM/R), thomson/Reuters (T/R), HSBC, and Barclays Bank PLC (BB). The range of each of these data sources differ, however if for an observation we have data from different sources we follow a priority rule. If data is available from World Markets Company/Reuters (WM/Reuters), we use that, otherwise we select from Thomson/Reuters, HSBC and Barclays Bank data sets in order. Following Hassan and Mano (2015), we also exclude observations for which covered interest parity seems to be violated.[46]

Our data set for macroeconomic indicators such as GDP and inflation comes from WorldBank. This includes all countries in the DataStream dataset except for Taiwan, and adds Finland, Greece, Portugal, and the U.S.

Our data for sovereign Credit Default Swaps comes from Datastream. Data stream has four different sovereign CDS series, for example, for Greece: 'GREECE SEN 1YR CDS - CDS PREM. MID','NAT BK OF GREECE SA SEN 1YR CDS - CDS PREM. MID','NAT BK OF GREECE SA SNR MM 1Y E - CDS PREM. MID', and 'NAT BK OF GREECE SA SNR MM 1Y $ - CDS PREM. MID'. For any particular month, Datastream has data on anywhere from zero to all four of these series. We compute a single CDS measure in the following way. We choose the first of these series as our baseline. We then regress this series on the second series for the entire pooled dataset (no fixed effects), this regression has an $R^2$ of 0.93. We use this regression to create estimates of series one based on series two. We repeat this by projecting series one on series three, the $R^2$ is 0.93; and series one on series four, the $R^2$ is 0.84. This gives us four partially overlapping CDS data series, all in the units of series one. We create a single CDS series by, within any single year, averaging over all of the available series out of the four.

[46]These observations are excluded from the sample and are as follow: Italy 1/1985 and 2/1985; Switzerland 2/1985; Germany 2/1985; United Kingdom 3/1985; Belgium 7/1990; Indonesia 12/1997, 3/1998, 5/1998-7/1998, 2/2001-11/2002, Austria 1/1990-2/1990; Spain 9/1987, 5/1988; Ireland 11/1986, 11/1987, 1/1989, 1/1991, 9/1992-11/1992, 1/1993; Belgium 2/1985; and Norway 2/1985. Also South Africa 8/1985 and Turkey before 11/2001 has been excluded following Lustig et al. (2011) due to large covered interest parity departures. Malaysia 8/1998-6/2005 and Indonesia 1/2003-5/2007 have also been removed because forward rates are zero.

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## Table 1: Empirical Results

In panel A we report the relationship between interest rate differentials and either currency appreciation (UIP), or the carry trade profit (CARRY). The right hand side variable is always $r_t^{rel} = log(R_t^{rel}) = log(R_t) - log(R_t^{US})$; the left hand side variable is either foreign currency appreciation against the U.S. dollar $-\Delta e_{t+1} = -log(E_{t+1}/E_t)$ or carry trade profit $r_{t+1}^C = -log(E_{t+1}/E_t) + log(R_t) - log(R_t^{US})$. In panel B we report the relationship between output growth and currency appreciation. The right hand side variable is always $gdp_t^{rel} = log(GDP_t^{rel}) = log(GDP_t) - log(GDP_t^{US})$; the left hand side variable is foreign currency appreciation against the U.S. dollar $-\Delta e_{t+1} = -log(E_{t+1}/E_t)$. Panel C is identical to Panel B but output growth is replaced by consumption growth. In Panel D we report the relationship between GDP growth and inflation. In the first line, the right and left hand side variables are GDP growth and inflation; in the second line they are GDP growth relative to the U.S., and inflation relative to the U.S. Panels A, B, C, and D report pooled regressions for an unbalanced panel of all countries (left), or summaries from separate regressions for each country (left). In country by country results, we report the average correlation, the average slope, the fraction of positive slopes (negative in panel D), and the fraction of significant slopes. Data for panels A, B, C, and D is annual 1982-2013 for 40 countries; it comes from Datastream and is described in detail in Appendix C. In panel E we report the relationship between GDP growth and inflation or TFP growth and inflation for the U.S. only; this data is quarterly 1947-2013 and comes from BEA.

| | Pooled | | | Country by Country | | | |
|---|---|---|---|---|---|---|---|
| | Corr | Slope | t-stat | E[Corr] | E[Slope] | %Slope | %t-stat |
| | Panel A: UIP and the Carry Trade $\mathcal{Y}_{t+1} = a + b \times r_t^{rel} + \epsilon_{t+1}$ | | | | | | |
| UIP: $\mathcal{Y} = -\Delta e$ | -0.064 | -0.179 | -1.521 | 0.058 | 0.263 | 55.3 | 7.9 |
| CARRY: $\mathcal{Y} = r^C$ | 0.280 | 0.821 | 6.968 | 0.313 | 1.263 | 92.1 | 31.6 |
| | Panel B: Balassa-Saumelson Effect $\mathcal{Y}_{t+1} = a + b \times \Delta gdp_{t+1}^{rel} + \epsilon_{t+1}$ | | | | | | |
| Nominal Exchange: $\mathcal{Y} = -\Delta e$ | 0.123 | 0.521 | 2.879 | 0.162 | 1.000 | 778 | 33.3 |
| Real Exchange: $\mathcal{Y} = -\Delta er$ | 0.087 | 0.351 | 2.201 | 0.131 | 0.856 | 65.7 | 20.0 |
| | Panel C: Backus-Smith Puzzle $\mathcal{Y}_{t+1} = a + b \times \Delta c_{t+1}^{rel} + \epsilon_{t+1}$ | | | | | | |
| Nominal Exchange: $\mathcal{Y} = -\Delta e$ | 0.124 | 0.514 | 2.614 | 0.185 | 1.024 | 77.8 | 19.4 |
| Real Exchange: $\mathcal{Y} = -\Delta er$ | 0.106 | 0.424 | 2.306 | 0.120 | 0.780 | 68.6 | 17.1 |
| | Panel D: GDP and Inflation | | | | | | |
| $i_{t+1} = a + b\Delta gdp_{t+1} + \epsilon_{t+1}$ | -0.051 | -0.160 | -1.841 | -0.073 | -0.344 | 47.4 | 18.4 |
| $i_{t+1}^{rel} = a + b\Delta gdp_{t+1}^{rel} + \epsilon_{t+1}$ | -0.101 | -0.308 | -2.666 | -0.182 | -0.506 | 76.3 | 26.3 |
| | Panel E: GDP and Inflation (USA) | | | | | | |
| $i_{t+1} = a + b\Delta gdp_{t+1} + \epsilon_{t+1}$ | -0.067 | -0.044 | -0.761 | | | | |
| $i_{t+1} = a + b\Delta tfp_{t+1} + \epsilon_{t+1}$ | -0.227 | -0.041 | -2.973 | | | | |

## Table 2: Carry Trade Characteristics

The top two panels shows the relation between interest rate differentials and country characteristics. In Panel A, the characteristics are future macroeconomic variables: the loading (beta) on world output, output growth, consumption growth, inflation, the volatility of output growth, the volatility of consumption growth, and the volatility of inflation. All quantities except for beta are relative to the same quantity computed for the U.S. (for example the volatility of inflation minus the volatility of U.S. inflation). The beta and volatilities are computed from annual growth for years $t+1$ through $t+4$ after the interest rate differential is observed; the growth rates are one year after the interest rate differential is observed. Although the interest rate differential is on the left hand side, since these are univariate regressions, these can be interpreted as predictive regressions because the right hand side variables are realized at $t+1$ or after while the interest rate differential is at $t$. In Panel B, the characteristics are the cost impeding financial trade from the U.S. into the target country (from Garleanu, Panageas, and Yu (2015), a country's size (measured as the log of its GDP relative to U.S. GDP), and the credit default spread of its sovereign debt (described in the text). All regressions in this panel are contemporaneous. This panel presents both univariate and multivariate regressions. In Panel C, we relate realized carry trade profits at $t+1$ to the interest rate differential at $t$, and to various country characteristics (some known at $t$ and some realized at $t+1$). The characteristics are output growth, consumption growth, and inflation realized at $t+1$, or costs impeding financial trade, a country's size, and its credit default spread known at $t$. The frequency is annual. All regressions are for an annual unbalanced panel of 40 countries from 1982 to 2013. The data is from DataStream and is described in the Appendix C.

Panel A: $r_t^{rel} = \gamma_0 + \gamma_x x$

| $x$ | $\beta^{\Delta y}_{t+1,t+4}$ | $\Delta y^{rel}_{t+1}$ | $\Delta c^{rel}_{t+1}$ | $i^{rel}_{t+1}$ | $\sigma^{\Delta y,rel}_{t+1,t+4}$ | $\sigma^{\Delta c,rel}_{t+1,t+4}$ | $\sigma^{i,rel}_{t+1,t+4}$ |
|---|---|---|---|---|---|---|---|
| $\gamma_x$ | 0.059 | -0.072 | 0.007 | 0.949 | 0.087 | 0.448 | 1.746 |
| t-stat | 0.652 | -1.077 | 0.085 | 7.507 | 0.695 | 2.576 | 2.666 |
| $R^2$ | 0.001 | 0.003 | 0.000 | 0.526 | 0.001 | 0.027 | 0.230 |
| $n$ | 619 | 705 | 644 | 682 | 619 | 556 | 596 |

Panel B: $r_t^{rel} = \gamma_0 + \gamma_x x$

| | | | | | |
|---|---|---|---|---|---|
| $\gamma_{COST}$ | 0.013 | | | | 0.001 |
| t-stat | 2.204 | | | | 1.488 |
| $\gamma_{SIZE}$ | | -0.031 | | -0.000 | -0.004 |
| t-stat | | -3.185 | | -0.007 | -1.458 |
| $\gamma_{CDS}$ | | | 0.010 | 0.010 | 0.006 |
| t-stat | | | 3.938 | 3.739 | 2.478 |
| $R^2$ | 0.044 | 0.014 | 0.161 | 0.161 | 0.215 |
| $n$ | 468 | 734 | 200 | 200 | 124 |

Panel C: $r_{t+1}^{C} = \gamma_0 + \gamma_r r_t^{rel} + \gamma_x x$

| $x$ | | $\Delta y_{t+1}$ | $\Delta c_{t+1}$ | $i_{t+1}$ | $COST_t$ | $SIZE_t$ | $CDS_t$ |
|---|---|---|---|---|---|---|---|
| $\gamma_r$ | 0.821 | 0.831 | 0.827 | 0.947 | 0.934 | 0.812 | -0.058 |
| t-stat | 6.968 | 7.129 | 7.259 | 6.054 | 6.835 | 6.750 | -0.140 |
| $\gamma_x$ | | 0.298 | 0.335 | -0.210 | -0.004 | -0.018 | 0.019 |
| t-stat | | 2.139 | 2.018 | -1.080 | -0.534 | -0.592 | 2.467 |
| $R^2$ | 0.079 | 0.083 | 0.087 | 0.082 | 0.106 | 0.079 | 0.035 |
| $n$ | 736 | 692 | 631 | 669 | 444 | 696 | 179 |

# Table 3: Model results

This table presents five groups of models each group has a different kind of asymmetry between countries H and F. In the first group (rows 1-4) F is more volatile than H ($\frac{\sigma_F}{\sigma_H} = 2$); in the second group (rows 5-8) F has higher risk aversion than H ($\frac{\gamma_F}{\gamma_H} = 2$); in the third group (rows 9-12) F is smaller than H; in the fourth group (rows 13-14) F-bonds default (this occurs in the worst state for both countries), within this group we present a case with zero inflation and a case with zero expected inflation; in the fifth group (rows 15-16) it is more costly to invest in F than in H (all investments are costless, except when H invests in F-bonds), within this group we present a case with zero inflation and a case with zero expected inflation. Columns 1-5 indicate key differences across models: available assets (Arrow-Debreu or Nominal Bonds), shocks (Non-Tradable or Tradable), monetary policy (Zero Inflation or Zero Expected Inflation), whether there is a trading cost, and whether default occurs. The remaining columns present the model's results, which are broken down into three categories: risk sharing, currency correlations, and currency returns. The log nominal exchange rate $e = log(E)$ indicates the strength of Home currency (Foreign currency per unit of Home), the log real exchange rate is defined as the nominal exchange rate multiplied by the price level: $er = log\left(E\frac{P_H}{P_F}\right)$. We define the log carry trade return as $r_{CT} = log\left(\frac{R_F}{R_H}\frac{E_0}{E_1}\right)$ regardless of whether F has a higher interest rate. This definition implies that the traditional carry trade profit is positive if interest differentials and $r_{CT}$ are both positive, or both negative. All expected returns and volatility are given as percent, ie 0.1 indicates 0.1%.

| Asset | Shock | Mon | Cost | Def | $c(\Delta c_H, \Delta c_F)$ | $\frac{\sigma(\Delta c_F)}{\sigma(\Delta c_H)}$ | $\sigma(\Delta er)$ | $c(z_H - z_F, \Delta er)$ | $\sigma(\Delta e)$ | $c(z_H - z_F, \Delta e)$ | $r_F - r_H$ | $\mathbb{E}[\Delta e]$ | $\mathbb{E}[r_{CT}]$ | $\sigma(r_{CT})$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel A: Asymmetry in volatility $\frac{\sigma_F}{\sigma_H} = 2$ | | | | | | | | | | | | | | |
| AD | NT | ZI | No | No | 0.929 | 1.316 | 7.539 | -1.000 | 7.539 | -1.000 | -0.468 | -0.256 | -0.724 | 7.539 |
| AD | T | ZI | No | No | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| NB | T | ZEI | No | No | 1.000 | 1.000 | 0.000 | 0.000 | 4.268 | 1.000 | 0.300 | 0.055 | 0.355 | 4.268 |
| NB | T | ZEI | $\kappa = 0.01$ | No | 0.650 | 1.723 | 2.252 | 1.000 | 6.523 | 1.000 | 0.299 | 0.239 | 0.538 | 6.523 |
| Panel B: Asymmetry in risk aversion $\frac{\gamma_F}{\gamma_H} = 2$ | | | | | | | | | | | | | | |
| AD | NT | ZI | No | No | 0.877 | 0.565 | 7.242 | -0.986 | 7.242 | -0.986 | -0.045 | -0.203 | -0.247 | 7.242 |
| AD | T | ZI | No | No | 1.000 | 0.545 | 1.190 | 0.000 | 1.190 | 0.000 | -0.005 | -0.158 | -0.163 | 1.190 |
| NB | T | ZEI | No | No | 1.000 | 1.000 | 0.000 | 0.000 | 4.045 | 1.000 | 0.021 | -0.021 | 0.000 | 4.045 |
| NB | T | ZEI | $\kappa = 0.01$ | No | 0.539 | 1.000 | 2.217 | 1.000 | 6.264 | 1.000 | 0.124 | 0.045 | 0.169 | 6.264 |
| Panel C: Asymmetry in size $\frac{Y_F}{Y_H} = 2$ | | | | | | | | | | | | | | |
| AD | NT | ZI | No | No | 0.906 | 0.898 | 7.145 | -1.000 | 7.145 | -1.000 | 0.144 | 0.130 | 0.275 | 7.145 |
| AD | T | ZI | No | No | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| NB | T | ZEI | No | No | 1.000 | 1.000 | 0.000 | 0.000 | 4.044 | 1.000 | -0.203 | -0.000 | -0.203 | 4.044 |
| NB | T | ZEI | $\kappa = 0.01$ | No | 0.552 | 0.882 | 2.214 | 1.000 | 6.262 | 1.000 | -0.260 | -0.077 | -0.337 | 6.262 |
| Panel D: Asymmetry in default, default in bad/bad state | | | | | | | | | | | | | | |
| NB | T | ZI | No | Yes | 0.109 | 0.795 | 3.651 | 0.929 | 3.651 | 0.929 | 3.462 | 0.142 | 0.408 | 8.404 |
| NB | T | ZEI | No | Yes | 0.356 | 0.678 | 2.891 | 0.780 | 6.556 | 0.961 | 3.250 | 0.293 | 0.346 | 9.592 |
| Panel E: Asymmetry in cost, $\kappa^{HF} = 0.01$, other $\kappa$'s are zero | | | | | | | | | | | | | | |
| NB | T | ZI | $\kappa^{HF} = 0.01$ | No | 0.420 | 1.000 | 2.585 | 1.000 | 2.585 | 1.000 | 0.113 | 0.169 | 0.282 | 2.585 |
| NB | T | ZEI | $\kappa^{HF} = 0.01$ | No | 0.836 | 1.000 | 1.208 | 1.000 | 5.255 | 1.000 | 0.107 | 0.161 | 0.268 | 5.255 |

# Figure 1: Exchange rate - SDF relationship

This figure plots the logarithm of the real exchange rate (Foreign currency per unit of Home) on the x-axis against the log of the ratio of the stochastic discount factors (Home divided by Foreign) on the y-axis, for selected models. For the cases of asymmetry in Volatility, Risk aversion, and Size, circles represent the complete market model with non-tradable shock (first row of Panels A, B, and C in Table 3) and triangles represent the incomplete markets model with costs (fourth row of Panels A, B, and C in Table 3). The figure titled Other Asymmetry, presents the incomplete market model with default, and the incomplete market model with financial trade costs; both have zero inflation (the first rows in Panels D and E of Table 3).